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SOCIAL SCIENCES

Performance Measurement and Best-Practice Benchmarking of Mutual Funds: Combining Stochastic Dominance criteria with Data Envelopment Analysis

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Background: SD efficiency tests & diversification

Kuosmanen, T. (2001): *Stochastic Dominance Efficiency Tests under Diversification*, Working Paper W-283, Helsinki School of Economics.

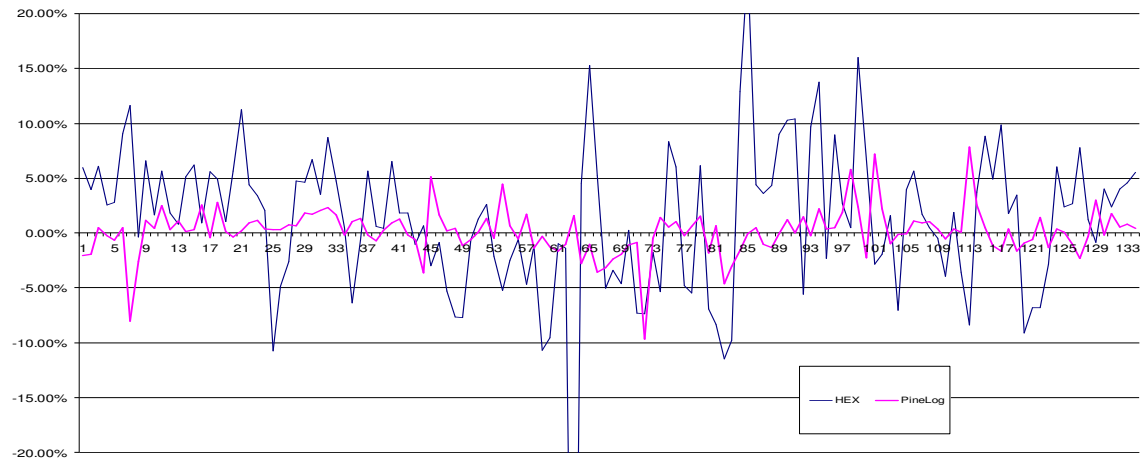
- First operational method to account for portfolio diversification within the SD framework
- The main idea: since we cannot diversify sorted return distributions, why not re-express SD criteria in terms of unsorted return vectors

PROBLEMS:

- Paper did not communicate well...

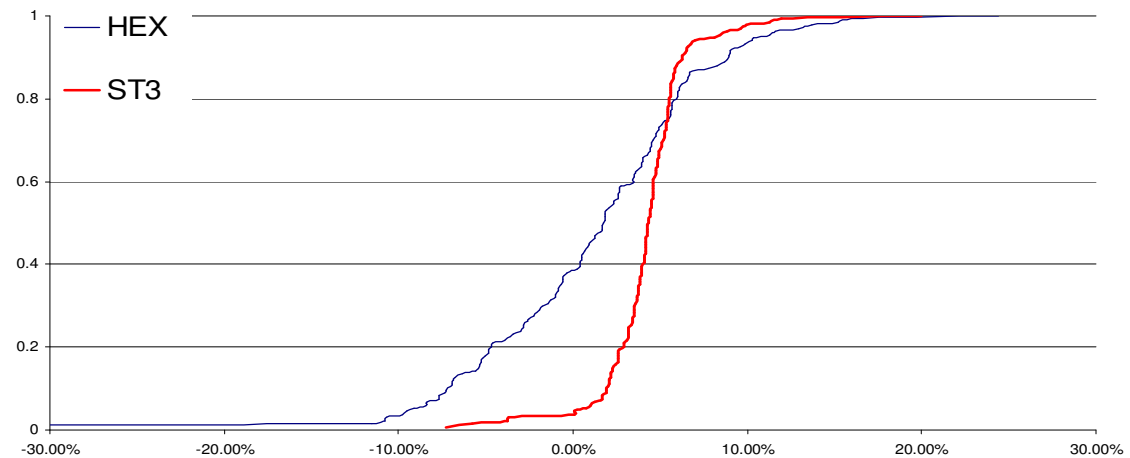
Main insight of Kuosmanen (2001) WP

1. Diversification
(time series)



2. Sorting / Ranking
(loss of information)

3. SD
(distribution function)



Background: SD efficiency tests & diversification

Post, G.T. (2003): Empirical Tests for Stochastic Dominance Efficiency, *Journal of Finance* 58(5), 1905-1931.

- Geared towards testing SSD efficiency of the market portfolio
- Emphasis on statistical inference
- Emphasis on computational simplicity to facilitate bootstrapping

Limitations:

- Necessary but not sufficient test
- Does not identify a dominating portfolio
- Does not measure the degree of efficiency
- Does not apply to FSD criterion

Background: SD efficiency tests & diversification

Kuosmanen, T. (2004): Efficient Diversification According to Stochastic Dominance Criteria, *Management Science* 50(10), 1390-1406.

(Revised and elaborated version of the 2001 WP)

- Shows why traditional crossing algorithms must fail to account for portfolio diversification
- Analytical expressions for FSD and SSD dominating sets
- Necessary and sufficient FSD and SSD tests
- Identifies a dominating benchmark portfolio
- Measures for the degree of inefficiency



Motivation: Two Approaches to Benchmarking

Benchmarking in management sciences:

- identification of industry best-practices
- “systematic comparison of elements of performance of an organization against those of other organizations, usually with the aim of mutual improvement” (Thor, 1996)

Motivation: Two Approaches to Benchmarking

Benchmarking in finance:

- Based on average performance
- geared towards rating and ranking of funds

Ansell, Moles, and Smart (2003): “Benchmarking of investment funds is rarely used to select the best product or organization. Instead it is employed to ensure that the product or organization meets a performance standard comparable to the rest of the population. It could be argued that the preferred strategy is to achieve close to the average performance, rather than to outperform the average.”



Objectives

- Present a new approach to mutual fund performance measurement based on best-practice benchmarking.
- Compare the mutual fund's performance with an endogenously selected benchmark portfolio that optimally tracks the evaluated fund's risk profile.
- Provide the fund managers, trustees and investors with information about efficiency improvement potential and identify portfolio strategies for achieving them.

Setting

- S states of nature
 - States randomly drawn without replacement such that each state occurs with the same probability
- Returns of the evaluated fund represented vector \mathbf{r}_0 .
- Model composes endogenously a dominating benchmark portfolio from stocks and other assets
 - Investment universe consists of N assets
 - $S \times N$ matrix of asset returns denoted by \mathbf{R}

Building blocks

■ Investment objectives

- Represented by absolute and stochastic dominance criteria
- Consistent with a wide range of investor preferences

■ Investment possibilities / constraints

- *Return possibilities set* modelled by Data Envelopment Analysis (DEA) (Farrell, 1957; Charnes et al., 1978)
- Restrictions on feasible portfolio weights

Building blocks

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- *Return possibilities set* modelled by Data Envelopment Analysis (DEA) (Farrell, 1957; Charnes et al., 1978)
- Restrictions on feasible portfolio weights

■ Efficiency indices

- Distance to the non-dominated boundary of return possibilities set
 - Additive Pareto-Koopmans (PK) measure (Charnes et al., 1985)
 - Directional distance function (DD) (Chambers et al., 1998)



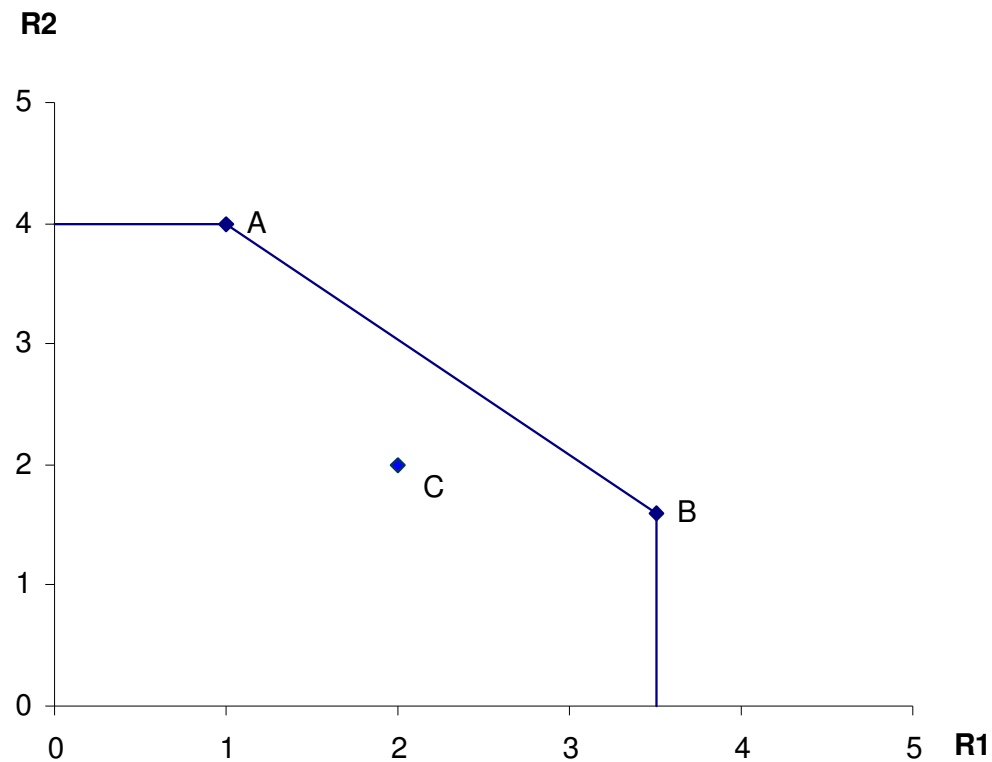
Return possibilities set

$$\mathcal{P} \equiv \{ \mathbf{r} \in \mathbb{R}^S \mid \mathbf{r} \leq \mathbf{R}\boldsymbol{\lambda}; \boldsymbol{\lambda} \in \Lambda \}$$

where $\boldsymbol{\lambda} \in \mathbb{R}^N$ is a vector of portfolio weights, and $\Lambda \subset \mathbb{R}^N$ is their feasible domain

Example: Return possibilities set, 2-periods

- Assets A, B, C; returns $\mathbf{r}_A=(1,4)$, $\mathbf{r}_B=(3.5,1.6)$, $\mathbf{r}_C=(2,2)$.



Absolute Dominance (AD) criterion

- Portfolio λ dominates mutual fund 0 in the sense of *absolute dominance* iff portfolio λ yields a return greater than or equal to that of mutual fund 0 in all states, and a strictly greater return in some state:

$$\mathbf{R}\lambda \geq \mathbf{r}_0 \quad \text{and} \quad \mathbf{R}\lambda \neq \mathbf{r}_0$$

First-order Stochastic Dominance (FSD)

The following conditions are equivalent:

- 1) Portfolio λ dominates mutual fund 0 by FSD.
- 2) $F_0(r) - F_\lambda(r) \geq 0$ with strict inequality for some r .
- 3) Every non-satiated decision maker prefers portfolio λ over mutual fund 0, with a strict preference for at least one such decision maker.
- 4) There exists a *permutation matrix* \mathbf{P} such that

$$\mathbf{R}\lambda \geq \mathbf{P}\mathbf{r}_0 \quad \text{and} \quad \mathbf{R}\lambda \neq \mathbf{P}\mathbf{r}_0$$

Second-order Stochastic Dominance (SSD)

The following conditions are equivalent:

- 1) Portfolio λ dominates mutual fund 0 by SSD.
- 2) $\int_0^r [F_0(s) - F_\lambda(s)] ds \geq 0$ with strict inequality for some r .
- 3) Every non-satiated, risk-averse decision maker prefers portfolio λ over mutual fund 0, with a strict preference for at least one such decision maker.
- 4) There exists a *doubly stochastic matrix* \mathbf{W} such that

$$\mathbf{R}\lambda \geq \mathbf{W}\mathbf{r}_0 \quad \text{and} \quad \mathbf{R}\lambda \neq \mathbf{W}\mathbf{r}_0$$

Illustration of the FSD dominating set

$\mathbf{r}_0 = (1,4)$.

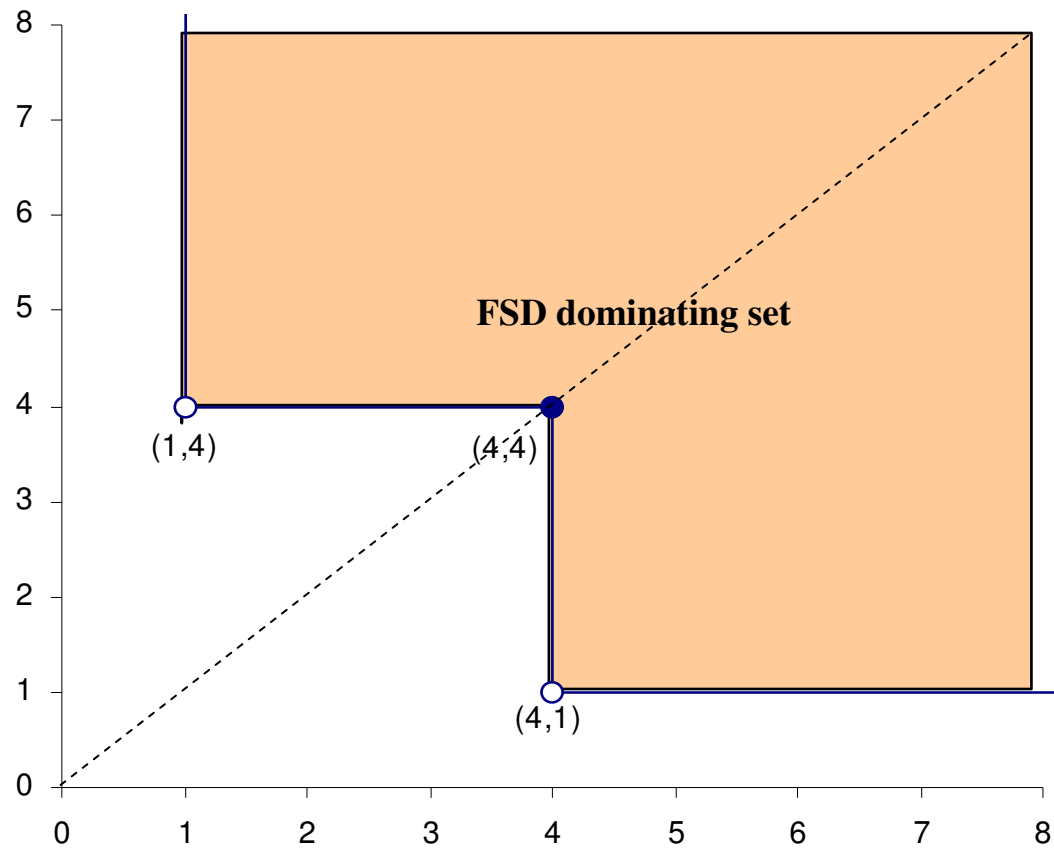
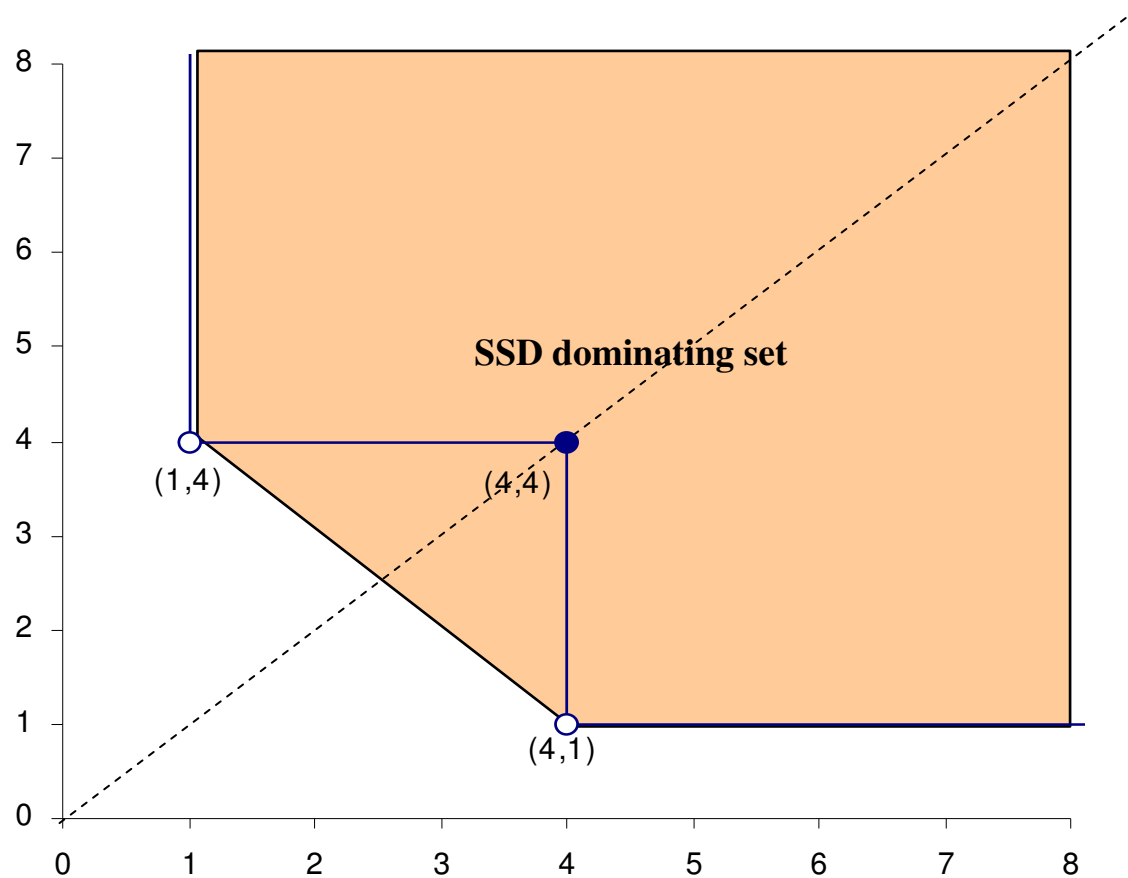


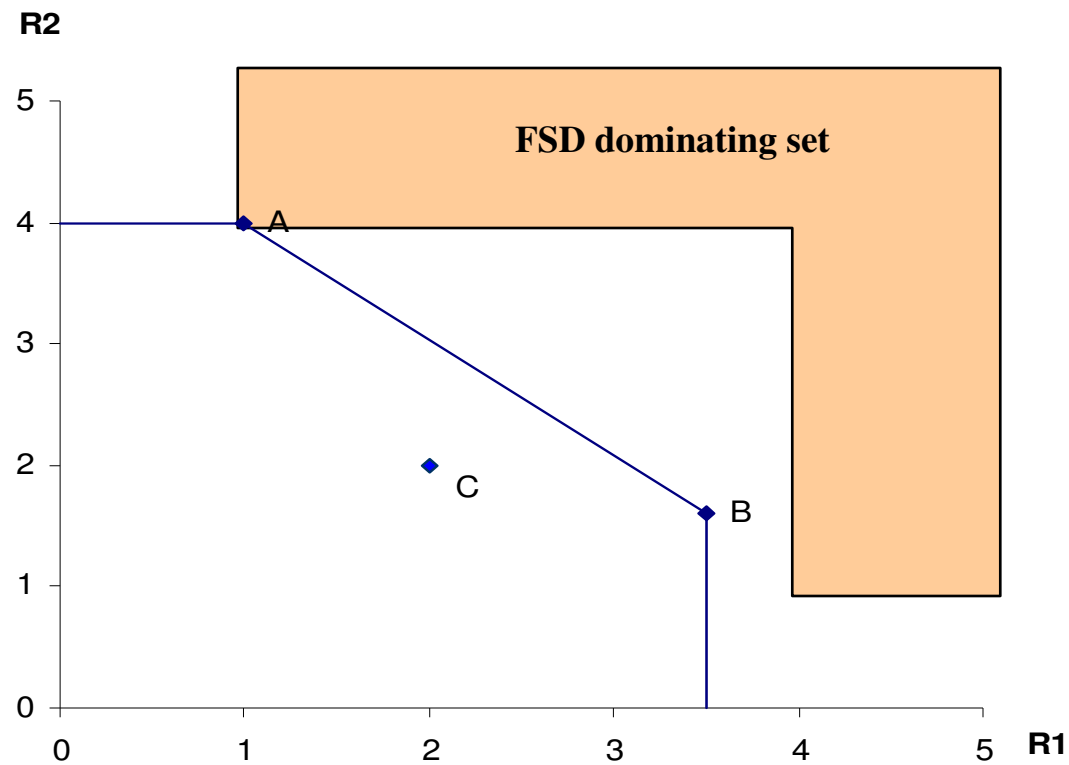
Illustration of the SSD dominating set

$\mathbf{r}_0 = (1,4)$.



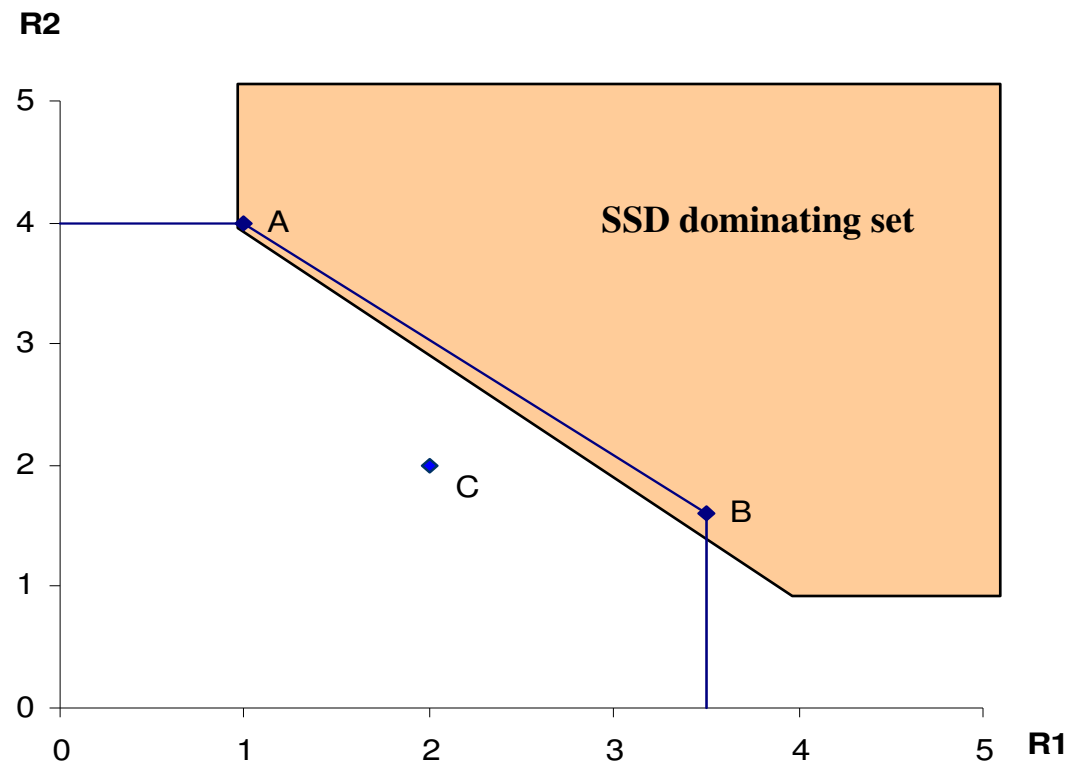
Combining SD sets with return possibilities

- Fund A is FSD efficient



Combining SD sets with return possibilities

- Fund A is SSD inefficient



Pareto-Koopmans (PK) efficiency indices

- Absolute dominance

$$PK^{AD}(\mathbf{r}_0) \equiv \max_{\mathbf{s} \geq \mathbf{0}} \{ \mathbf{1}'\mathbf{s} \mid \mathbf{r}_0 + \mathbf{s} \in \mathcal{P} \}$$

- First-order Stochastic Dominance

$$PK^{FSD}(\mathbf{r}_0) \equiv \max_{\mathbf{s} \geq \mathbf{0}, \mathbf{P} \in \Pi} \{ \mathbf{1}'\mathbf{s} \mid \mathbf{P}\mathbf{r}_0 + \mathbf{s} \in \mathcal{P} \}$$

- Second-order Stochastic Dominance

$$PK^{SSD}(\mathbf{r}_0) \equiv \max_{\mathbf{s} \geq \mathbf{0}, \mathbf{W} \in \Xi} \{ \mathbf{1}'\mathbf{s} \mid \mathbf{W}\mathbf{r}_0 + \mathbf{s} \in \mathcal{P} \}$$

Pareto-Koopmans (PK) efficiency indices

Economic interpretation

- $PK(.) / S$ measures the potential increase in the mean return achievable with the current or more favorable risk exposure of the fund
- Note: if $PK(.) > 0$, then the benchmark portfolio λ^* dominates the mutual fund 0 (in the sense of AD, FSD, or SSD).

Some properties of the PK efficiency indices

- Always non-negative
 - Problem is infeasible if \mathbf{r}_0 not contained in the return possibilities set
- $PK^{AD}(\cdot)=0$ and $PK^{FSD}(\cdot)=0$ are both necessary and sufficient efficiency conditions
- All indices are monotonic increasing functions of \mathbf{r}_0
 - Important for consistency of efficiency rankings
- $PK^{AD}(\cdot)$ and $PK^{SSD}(\cdot)$ are continuous functions of \mathbf{r}_0
 - Important for stability of the efficiency indices

Primal LP/MILP formulations

- Absolute dominance

$$PK^{AD}(\mathbf{r}_0) = \max_{\mathbf{s}, \lambda} \{ \mathbf{1}'\mathbf{s} \mid \mathbf{r}_0 + \mathbf{s} = \mathbf{R}\lambda; \mathbf{s} \geq \mathbf{0}; \lambda \in \Lambda \}$$

- First-order Stochastic Dominance

$$PK^{FSD}(\mathbf{r}_0) = \max_{\mathbf{s}, \lambda, \mathbf{P}} \{ \mathbf{1}'\mathbf{s} \mid \mathbf{P}\mathbf{r}_0 + \mathbf{s} = \mathbf{R}\lambda; \mathbf{s} \geq \mathbf{0}; \lambda \in \Lambda; \mathbf{P} \in \Pi \}$$

- Second-order Stochastic Dominance

$$PK^{SSD}(\mathbf{r}_0) = \max_{\mathbf{s}, \lambda, \mathbf{W}} \{ \mathbf{1}'\mathbf{s} \mid \mathbf{W}\mathbf{r}_0 + \mathbf{s} = \mathbf{R}\lambda; \mathbf{s} \geq \mathbf{0}; \lambda \in \Lambda; \mathbf{W} \in \Xi \}$$

Dual formulations

- Absolute dominance

$$PK^{AD}(\mathbf{r}_0) = \min_{\phi, \mathbf{v}} \{ \phi \mid \phi \mathbf{1} \geq \mathbf{v}'(\mathbf{R} - \mathbf{r}_0 \mathbf{1}'); \mathbf{v} \geq \mathbf{1} \}$$

- First-order Stochastic Dominance

-use the property $PK^{FSD}(\mathbf{r}_0) = \max_{\mathbf{P} \in \Pi} PK^{AD}(\mathbf{P}\mathbf{r}_0)$

- Second-order Stochastic Dominance

$$PK^{SSD}(\mathbf{r}_0) = \min_{\beta, \boldsymbol{\theta}, \boldsymbol{\tau}, \mathbf{v}} \{ \beta - (\mathbf{1}'\boldsymbol{\theta} + \mathbf{1}'\boldsymbol{\tau}) \mid \beta \mathbf{1} \geq \mathbf{v}'\mathbf{R}; v_s r_{0t} \geq \theta_t + \tau_s \quad \forall t, s \in \sigma; \mathbf{v} \geq \mathbf{1} \}$$

Economic interpretation of the dual

- Vector \mathbf{v} characterizes a tangent hyperplane to the return possibilities set
- In the FSD and SSD cases, if multipliers \mathbf{v} are interpreted as average utilities $v_s = u(r_{0s}) / r_{0s}$, then the expected utility of fund 0 can be expressed as $\mathbf{v}'\mathbf{r}_0 / S$.
- Thus, PK inefficiency can be interpreted as a minimum loss of expected utility
- Using vector \mathbf{v} , one could derive bounds for the class of utility functions that can rationalize an efficient portfolio, following Varian (1983).

Directional distance functions (DD)

- Absolute dominance

$$DD^{AD}(\mathbf{r}_0) \equiv \max_{\delta} \{ \delta \mid \mathbf{r}_0 + \delta \mathbf{g} \in \mathcal{P} \}$$

- First-order Stochastic Dominance

$$DD^{FSD}(\mathbf{r}_0) \equiv \max_{\delta, \mathbf{P} \in \Pi} \{ \delta \mid \mathbf{P} \mathbf{r}_0 + \delta \mathbf{g} \in \mathcal{P} \}$$

- Second-order Stochastic Dominance

$$DD^{SSD}(\mathbf{r}_0) \equiv \max_{\delta, \mathbf{W} \in \Xi} \{ \delta \mid \mathbf{W} \mathbf{r}_0 + \delta \mathbf{g} \in \mathcal{P} \}$$

Directional distance functions (DD)

Economic interpretation

- Depends on the direction vector \mathbf{g} .
- Setting $\mathbf{g} = \mathbf{1}$, we can interpret $DD(.)$ as the minimum risk free premium that must be added to \mathbf{r}_0 to make the fund efficient (in the sense of AD, FSD, or SSD).

Some properties of the DD measure

- Can be positive or negative
 - Negative values indicate “super-efficiency”: \mathbf{r}_0 not contained in the return possibilities set
- Do not offer necessary and sufficient efficiency conditions
- Monotonic increasing functions of \mathbf{r}_0
- $DD^{AD}(\cdot)$ and $DD^{SSD}(\cdot)$ are continuous functions of \mathbf{r}_0

Primal LP/MILP formulations

- Absolute dominance

$$DD^{AD}(\mathbf{r}_0) = \max_{\delta, \lambda} \{ \delta \mid \mathbf{r}_0 + \delta \mathbf{g} = \mathbf{R}\lambda; \lambda \in \Lambda \}$$

- First-order Stochastic Dominance

$$DD^{FSD}(\mathbf{r}_0) = \max_{\delta, \lambda, \mathbf{P}} \{ \delta \mid \mathbf{P}\mathbf{r}_0 + \delta \mathbf{g} = \mathbf{R}\lambda; \lambda \in \Lambda; \mathbf{P} \in \Pi \}$$

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Dual formulations

- Absolute dominance

$$DD^{AD}(\mathbf{r}_0) = \min_{\delta, \mathbf{v}} \{ \delta \mid \delta \mathbf{1} \geq \mathbf{v}'(\mathbf{R} - \mathbf{r}_0 \mathbf{1}'); \mathbf{v}'\mathbf{g} = 1 \}$$

- First-order Stochastic Dominance

-use the property $DD^{FSD}(\mathbf{r}_0) = \max_{\mathbf{P} \in \Pi} DD^{AD}(\mathbf{P}\mathbf{r}_0)$

- Second-order Stochastic Dominance

$$DD^{SSD}(\mathbf{r}_0) = \min_{\beta, \boldsymbol{\theta}, \boldsymbol{\tau}, \mathbf{v}} \{ \beta - (\mathbf{1}'\boldsymbol{\theta} + \mathbf{1}'\boldsymbol{\tau}) \mid \mathbf{v}'\mathbf{g} = 1; \beta \mathbf{1} \geq \mathbf{v}'\mathbf{R}; v_s r_{0t} \geq \theta_t + \tau_s \quad \forall t, s \in \sigma; \mathbf{v} \geq \mathbf{1} \}$$

Illustrative application

Selection criteria for mutual funds:

- 1) Applies a *positive screen* to environmental criteria (i.e., seeks companies with a positive record in terms of the environment).
- 2) Applies an *exclusionary screen* to environmental criteria (avoids companies with a poor record in terms of the environment).
- 3) The shares traded in the NYSE since January 1, 2000 or earlier.
- 4) Is a large-blend equity fund following growth strategy.

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→ Homogenous group of 8 mutual funds

Return possibilities set

- 175 stocks traded in NYSE and included in the DJSI sustainability index
- Weekly returns for 26/11/2001 - 26/11/2002
- Constraints on portfolio weights
 - no shortsales
 - weight of any single stock should not exceed 5.8%
 - weight of bonds should not exceed 5.8%
 - weight of the large cap. US stocks at least 65%



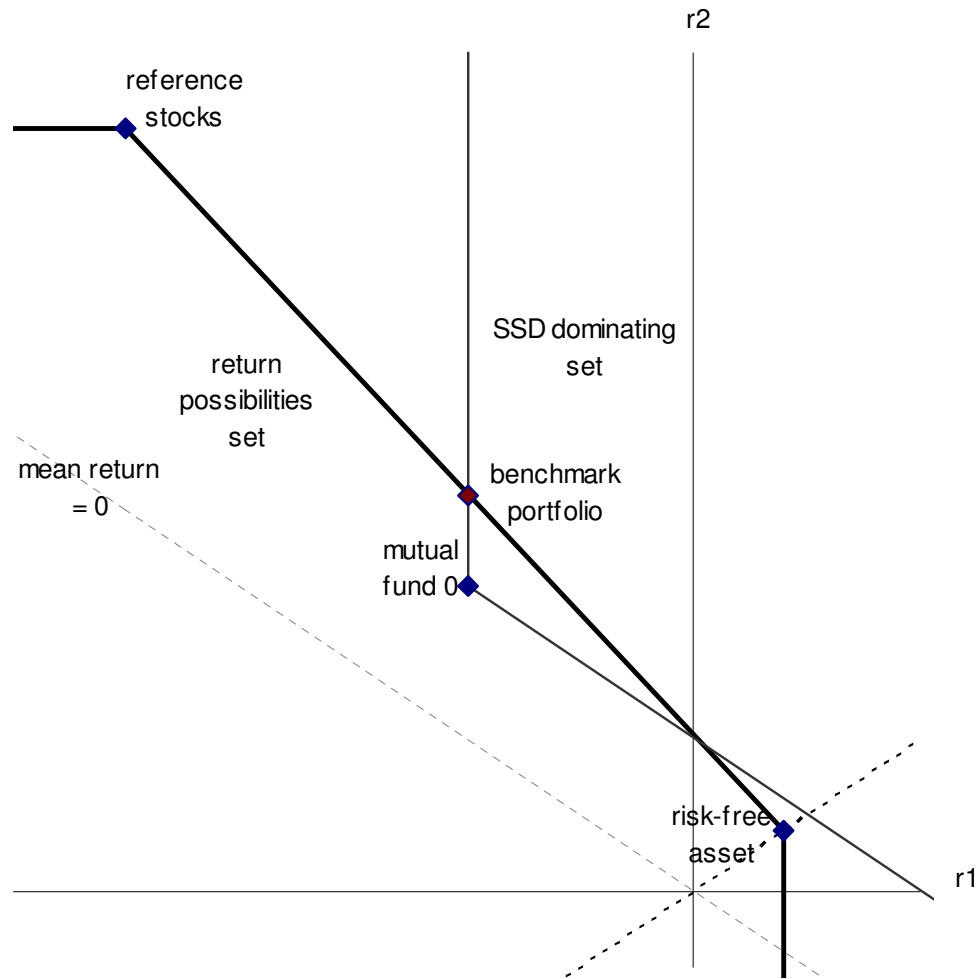
Descriptive statistics of returns

	Mean	St.dev.	Min	Max
Mutual funds	-0.179	1.449	-6.494	3.289
Stocks	-0.138	3.154	-33.481	35.360

Efficiency measures (inefficiency % points p.a.)

Mutual fund (ticker symbol)	<i>PK / S</i> measure			<i>DD (g = 1)</i> measure		
	AD	FSD	SSD	AD	FSD	SSD
CSIEX	0.000	0.350	0.350	0.000	0.349	0.349
CSECX	0.000	0.358	0.358	0.000	0.357	0.357
FEMMX	0.000	0.364	0.364	0.000	0.363	0.363
NBSRX	0.000	0.432	0.432	0.000	0.432	0.432
DESRX	0.237	0.433	0.433	0.014	0.432	0.432
ADVOX	0.368	0.454	0.454	0.102	0.454	0.454
GCEQX	0.000	0.485	0.485	0.000	0.484	0.484
DSEFX	0.000	0.507	0.507	0.000	0.506	0.506
Average	0.076	0.423	0.423	0.015	0.422	0.422
St. Dev.	0.144	0.060	0.060	0.036	0.060	0.060

Why FSD and SSD measures are same?

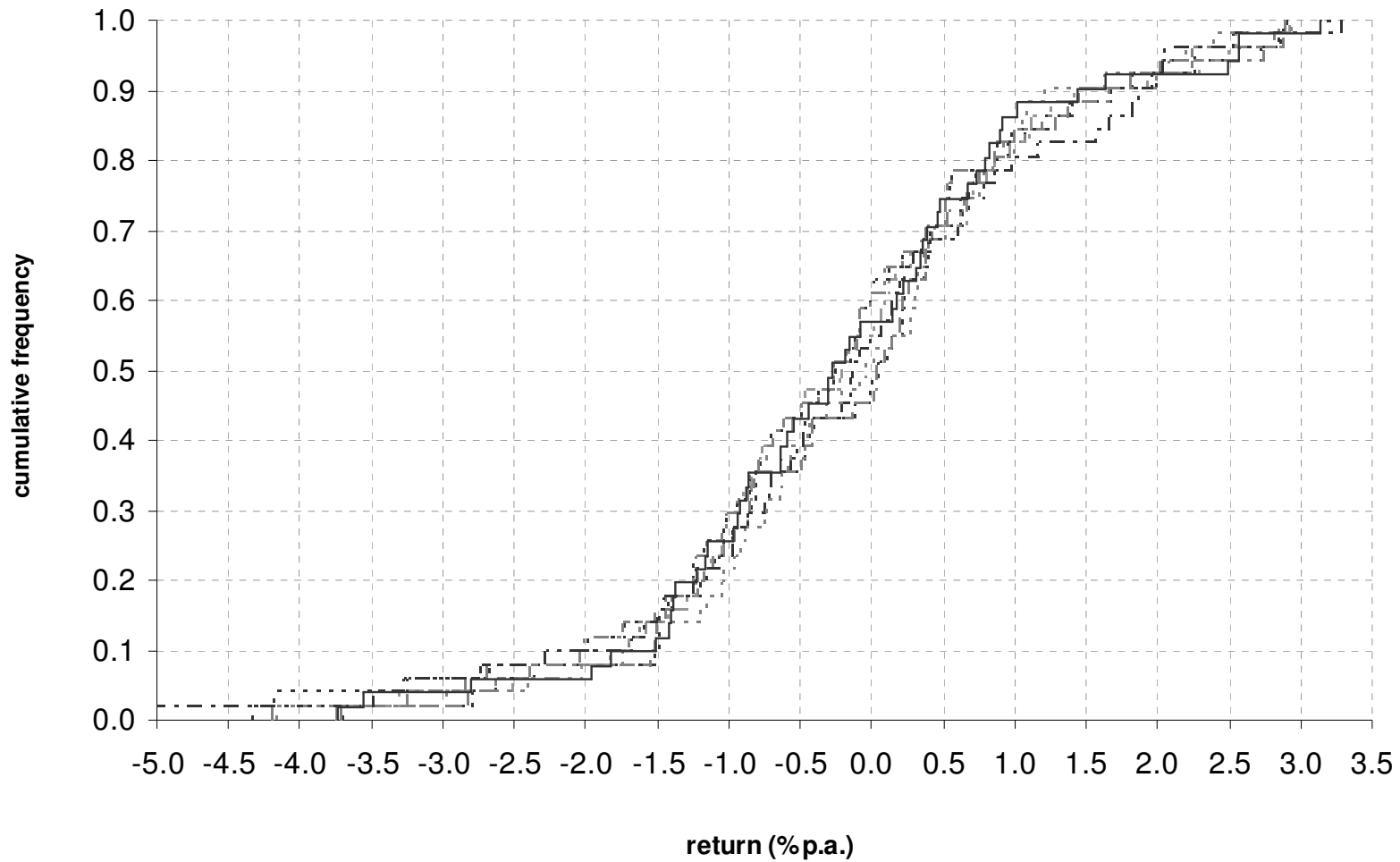


Benchmark portfolios

Company name	Industry	average	st.dev
Alcoa Inc.	Aluminum	0.058	0
Amcor Ltd	Packaging & Containers	0.058	0
Dell Inc.	Personal Computers	0.058	0
Eastman Kodak	Photographic Equipment & Supplies	0.058	0
ENSCO International Inc	Oil & Gas Drilling & Exploration	0.058	0
Entergy Corp.	Electric Utilities	0.058	0
Johnson & Johnson	Drug Manufacturing	0.058	0
Mattel, Inc.	Toys & Games	0.058	0
Mentor Corp.	Medical Appliances & Equipment	0.058	0
Royal Caribbean Cruises Ltd.	Entertainment	0.058	0
Safeway Inc.	Food retail	0.058	0
SKF	Industrial Goods & Services	0.058	0
Boeing	Aerospace & Defense	0.058	0
United Health Group Inc	Health Care Plans	0.058	0
Visteon Corp.	Auto Parts	0.058	0
Canon Inc.	Photographic Equipment & Supplies	0.036	0.016
Intel Corp.	Semiconductors	0.034	0.016
ANZ Banking Group Ltd	Banking	0.033	0.023
Cognos Inc	Application Software	0.031	0.022
BHP Billiton Ltd	Industrial Metals & Minerals	0.001	0.001

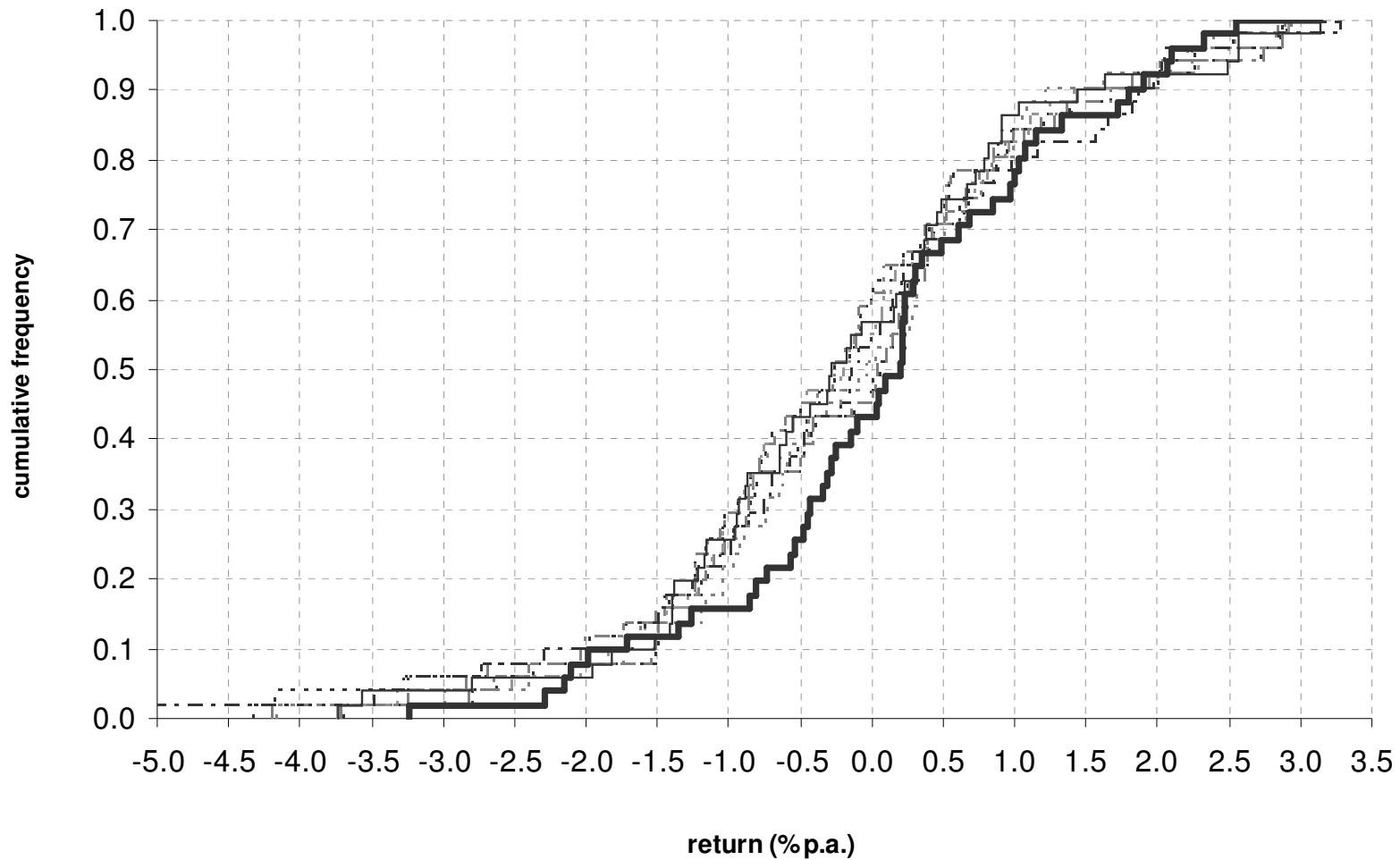


Cumulative return distributions – 8 funds





Cumulative return distributions – 8 funds



Summary

- Established links between the AD criteria and SD.
- Examined two alternative efficiency metrics:
 - the slack-based Pareto-Koopmans measure (Charnes et al. 1985)
 - the directional distance function (Chambers et al. 1998)
- Derived dual expressions for the efficiency measures, and discussed their utility theoretic interpretations.
- Discussed some practical issues including the specification of benchmark units, preprocessing of data, statistical testing of normality hypothesis, and the interpretation and illustration of the results.

Thank you for your attention!

- Full paper available from the author by request
- Questions and comments are welcome
- Contact:
 - E-mail: Timo.Kuosmanen@wur.nl
 - Homepage: <http://www.sls.wau.nl/enr/staff/kuosmanen>