Neoclassical versus frontier production models?

Testing for the skewness
of regression residuals

Timo Kuosmanen
MTT & HSE

Mogens Fosgerau
Technical University of Denmark

Empirical models of production

- Neoclassical models
  - focus on the production / cost function and its properties (substitution / scale elasticities)
  - full efficiency or average practice
  - no particular attention on residuals

- Frontier models
  - production / cost function represents the best practice
  - main focus on residuals, consisting of inefficiency and noise components

Neoclassical model

$$y_i = f(\mathbf{x}_i) + \varepsilon_i, \quad i = 1,...,n$$

y_i is (log-)output of firm *i*

f is production function

$\mathbf{x}_i$ is vector of (log-)inputs

ε_i is disturbance term

$$E(\varepsilon_i \middle| \mathbf{x}_1,...,\mathbf{x}_n) = 0 \;\; \forall i = 1,...,n$$

Frontier model

$$y_i = f(\mathbf{x}_i) + \varepsilon_i, \quad i = 1, \ldots, n$$

$$\varepsilon_i = v_i - u_i, \quad i = 1, \ldots, n$$

y_i is (log-)output of firm *i*

f is production function

$\mathbf{x}_i$ is vector of (log-)inputs

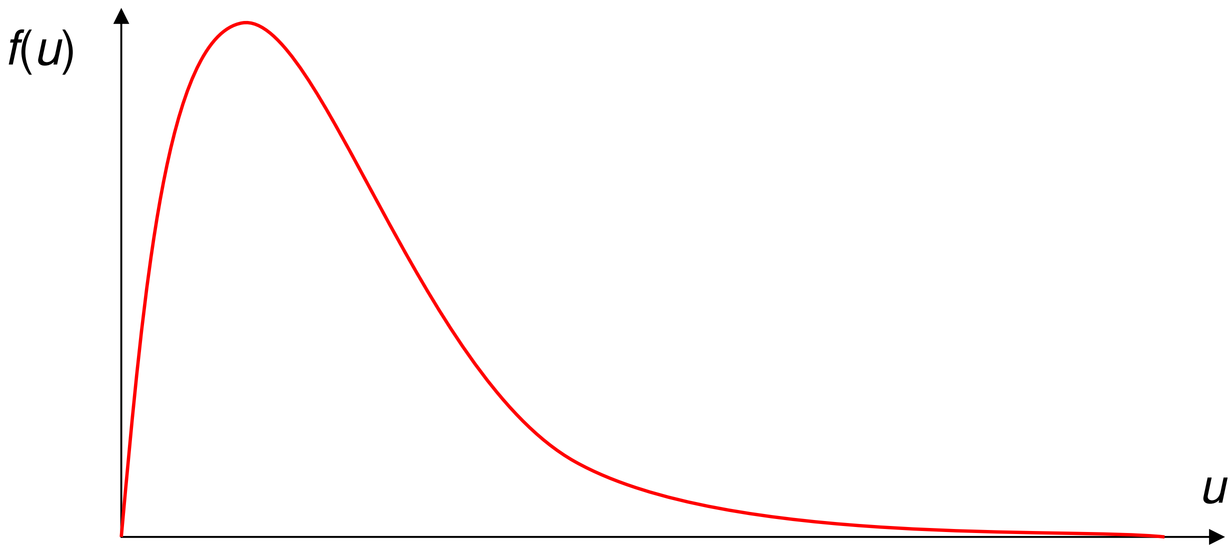
ε_i is disturbance term

v_i is symmetric noise term

$u_i \geq 0$ is asymmetric inefficiency term

Neoclassical vs. frontier model

- Negative skewness of the disturbance ε is the key testable hypothesis
- Neoclassical model is a special case if u_i=0 or if u_i has symmetric distribution



Neoclassical vs. frontier model

- Negative skewness of the disturbance ε is the key testable hypothesis
- Neoclassical model is a special case if u_i=0 or if u_i have a symmetric distribution

- Note: $E(y_i|\mathbf{x}_i) = f(\mathbf{x}_i) - E(u_i) < f(\mathbf{x}_i)$

  if asymmetric u_i is present in ε, but it is ignored, estimating the neoclassical model yields biased and inconsistent results.

Why inefficiency is asymmetric?

- Suppose inefficiency *u* is a function of unobserved management skill *s*: $s \sim N(\mu,\sigma^2)$

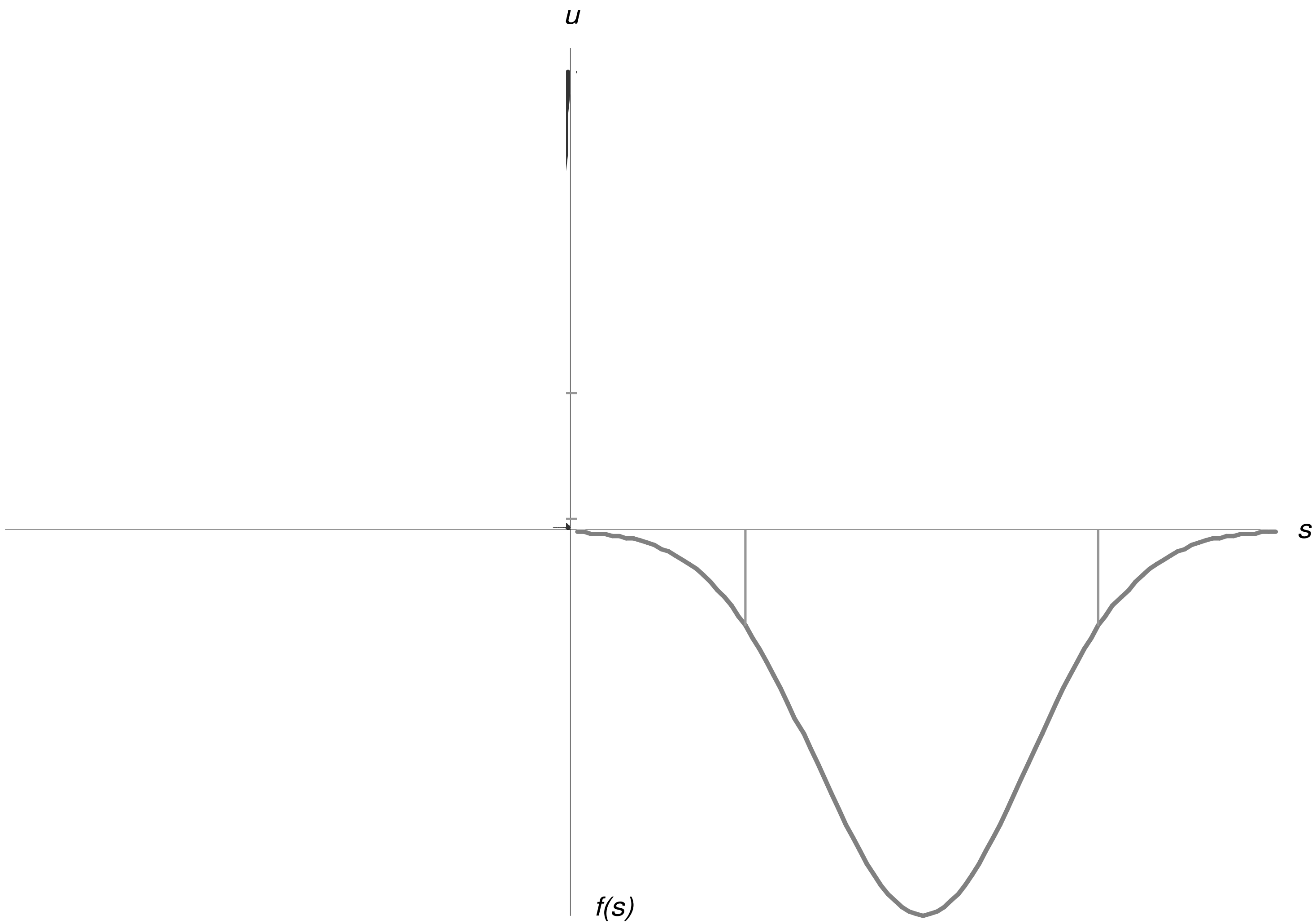
- Assumptions:

1) Minimum inefficiency is zero: $\lim_{s \to \infty} u(s) = 0$

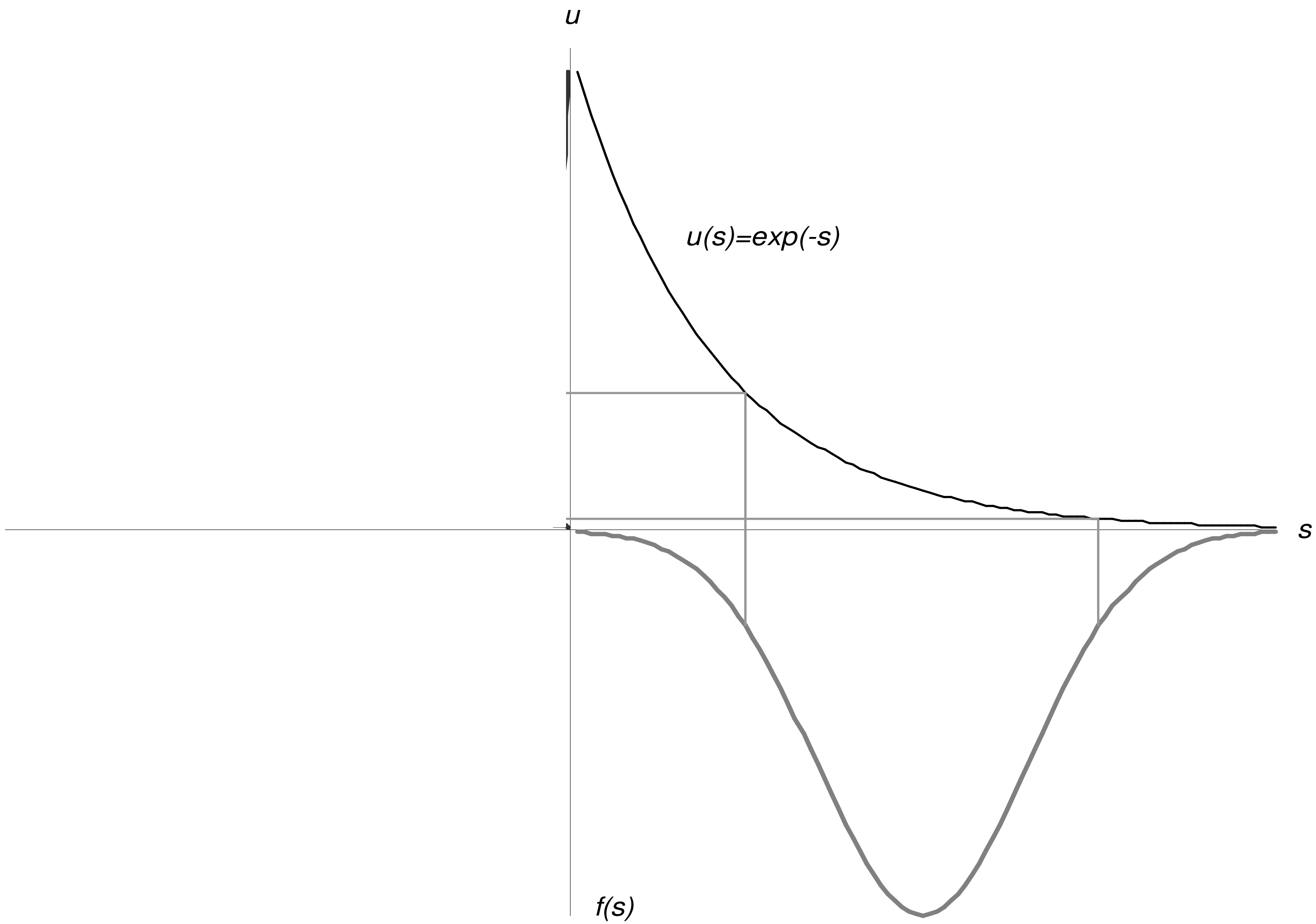
2) Inefficiency decreases as skill increases: $u'(s) < 0$ for all s

3) Achieving higher levels of performance gets progressively more difficult: skill has diminishing marginal returns $u''(s) > 0$

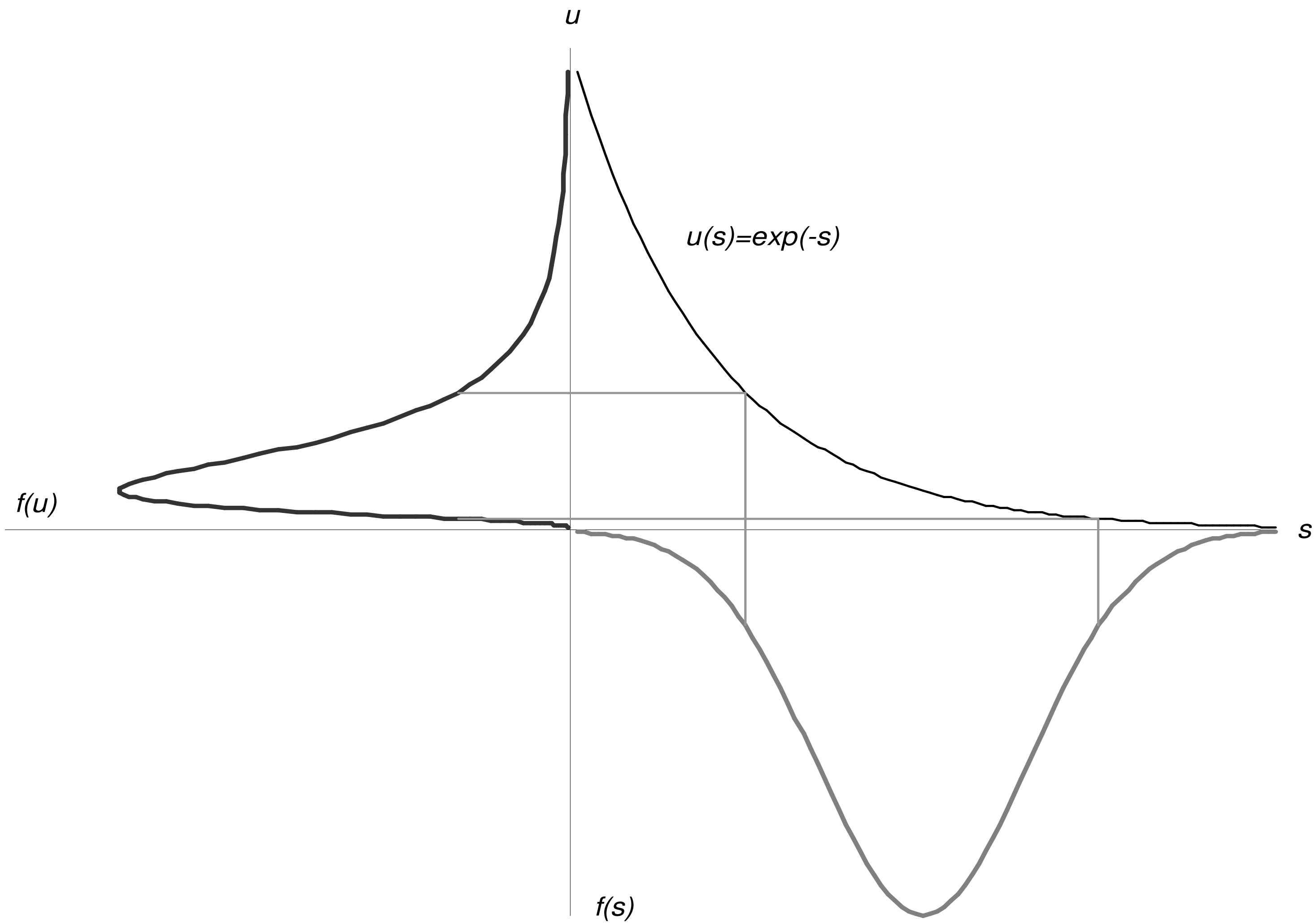
Example: $s \sim N(\mu,\sigma^2)$



Example: $u(s) = \exp(-s)$



Example: $u \sim \ln N(\mu,\sigma^2)$



Specification test

- We can tests if skill influences performance directly, or through a convex inefficiency function $u(s)$ that causes asymmetry

Step 1: Estimate the conditional expectation $E(y_i|\mathbf{x}_i) = f(\mathbf{x}_i) - E(u_i)$ by parametric or nonparametric regression techniques

Step 2: Given residuals $\{e_1,\ldots,e_n\}$ from Step 1, test for skewness of residuals.

Hypotheses to be tested

- Neoclassical model is a restricted special case of the frontier model when **u**=0

H_0: Neoclassical model is correct specification

H_1: Frontier model is correct specification

- Power of the test depends on the specific definition of H_0 and H_1.

Hypotheses to be tested

- Two interpretations:

$\mathbf{H_0}$: disturbances **ε** are normally distributed

$\mathbf{H_1}$: disturbances **ε** exhibit non-normal negative skewness

$\mathbf{H_0}$´: disturbances **ε** are symmetrically distributed

$\mathbf{H_1}$´: disturbances **ε** exhibit asymmetric negative skewness

Testing for normality against skewness

- Several tests are available (e.g., Shapiro-Wilk, D'Agostino-Pearson, and Jarque-Bera tests)
- Our preferred choice: 3rd sample moment test (Shapiro, Wilk, Chen 1968)
- Test statistic

$$\sqrt{b_1} = m_3 / (m_2)^{3/2}$$

- The critical values of the test statistic under H_0 obtained by Monte Carlo simulations

Testing for normality against skewness

- Problem: tests may reject H_0 even if the true distribution is perfectly symmetric but has non-normal kurtosis
  - Fat or thin tails in residual distribution difficult to interpret as evidence in favor of frontier model

- Solution: test also for normality against kurtosis

Testing for normality against kurtosis

- 4th sample moment test (D'Agostino and Pearson, 1973)
- Test statistic

$$b_2 = m_4 / (m_2)^2$$

- The critical values of the test statistic under H_0 obtained by Monte Carlo simulations

Interpretation

	Kurtosis insignificant	Kurtosis significant
Skewness insignificant	Neoclassical model	Neoclassical (with caution)
Skewness significant	Frontier model	Inconclusive -> test for symmetry

Example application

- U.S. electricity companies
- Two cross-sections in years
  1955 (n=145) and 1970 (n=123)
  - Nerlove (1963)
  - Christensen and Greene (1976, *JPE*)

- Output: electricity
- Inputs: labor, capital, fuel

Results

	Production function	test statistics		p-values		
year	regression method	$\sqrt{b_1}$	b_2	$\sqrt{b_1}$ Norm.	b_2 Norm	$\sqrt{b_1}$ Sym
1955	OLS Cobb-Douglas	-0.209	8.287	0.255	**0.000**	0.448
	OLS Translog	-0.530	7.630	**0.044**	**0.000**	0.339
	Kernel	-2.356	8.052	**0.000**	**0.000**	**0.007**
	Nonparametric least squares	-0.263	9.194	0.203	**0.000**	0.437
1970	OLS Cobb-Douglas	-1.256	8.127	**0.000**	**0.000**	0.195
	OLS Translog	-0.092	5.509	0.395	**0.000**	0.457
	Kernel	-3.835	18.511	**0.000**	**0.000**	**0.047**
	Nonparametric least squares	-2.045	10.029	**0.000**	**0.000**	0.089

Conclusions

- It is no longer necessary to choose a neoclassical or frontier model based on faith: it is ultimately an empirical question.
- Negative skewness of disturbances identified as the testable hypothesis of the frontier model
- Developed a theoretical model and statistical specification tests

Further research

- Besides production / cost functions, skewness of regression residuals potentially interesting in other areas as well

- Examples:
  - Elections: what factors explain candidates' success (votes)?
  - Labor market matching: which factors contribute to efficient matching of job seekers to vacancies?

Thank you!

- Questions and comments are welcome
- E-mail: timo.kuosmanen@mtt.fi