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Towards One Stage Non- / Semiparametric Estimation and a Unified Framework

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DEA as nonparametric least squares

Model

$$y = f(\mathbf{x}) - u$$

f monotonic and concave

$$u \geq 0$$

Estimator

$$\min \sum_{i=1}^{n} u_i^2$$

$$s.t.$$

$$y_i = \alpha_i + \boldsymbol{\beta}_i' \mathbf{x}_i - u_i \quad \forall i$$

$$\alpha_i + \boldsymbol{\beta}_i' \mathbf{x}_i \leq \alpha_h + \boldsymbol{\beta}_h' \mathbf{x}_i \quad \forall h, i$$

$$\boldsymbol{\beta}_i \geq \mathbf{0} \quad \forall i$$

$$u_i \geq 0 \quad \forall i$$

Kuosmanen & Johnson (2010) Data Envelopment Analysis as Nonparametric Least Squares Regression, *Operations Research.*

DEA with z as inputs

Model

$$y = f(\mathbf{x}, \mathbf{z}) - u$$

f monotonic and concave

$$u \geq 0$$

Estimator

$$\min \sum_{i=1}^{n} u_i^2$$

$$s.t.$$

$$y_i = \alpha_i + \boldsymbol{\beta}_i' \mathbf{x}_i + \boldsymbol{\delta}_i' \mathbf{z}_i - u_i \quad \forall i$$

$$\alpha_i + \boldsymbol{\beta}_i' \mathbf{x}_i + \boldsymbol{\delta}_i' \mathbf{z}_i \leq \alpha_h + \boldsymbol{\beta}_h' \mathbf{x}_i + \boldsymbol{\delta}_h' \mathbf{z}_i \quad \forall h, i$$

$$\boldsymbol{\beta}_i \geq \mathbf{0} \quad \forall i$$

$$\boldsymbol{\delta}_i \geq \mathbf{0} \quad \forall i$$

$$u_i \geq 0 \quad \forall i$$

One stage DEA

Model

$$y = f(\mathbf{x}) + \boldsymbol{\delta}'\mathbf{z} - u$$

f monotonic and concave

$u \geq 0$

Estimator

$$\min \sum_{i=1}^{n} u_i^2$$

$s.t.$

$$y_i = \alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_i + \boldsymbol{\delta}'\mathbf{z}_i - u_i \quad \forall i$$

$$\alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_i \leq \alpha_h + \boldsymbol{\beta}_h'\mathbf{x}_i \quad \forall h, i$$

$$\boldsymbol{\beta}_i \geq \mathbf{0} \quad \forall i$$

$$u_i \geq 0 \quad \forall i$$

Johnson & Kuosmanen (2009) How Operational Conditions and Practices Affect Productive Performance? Efficient Semi-Parametric One-Stage Estimators, SSRN working paper. http://ssrn.com/abstract=1485733

StoNEZD

Model

$$y = f(\mathbf{x}) + \boldsymbol{\delta}'\mathbf{z} + \varepsilon$$

f monotonic and concave

$$\varepsilon = v - u$$

v symmetric noise

$u \geq 0$ random inefficiency

Estimator

$$\min \sum_{i=1}^{n} \varepsilon_i^2$$

$s.t.$

$$y_i = \alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_i + \boldsymbol{\delta}'\mathbf{z}_i + \varepsilon_i \;\; \forall i$$

$$\alpha_i + \boldsymbol{\beta}_i'\mathbf{x}_i \leq \alpha_h + \boldsymbol{\beta}_h'\mathbf{x}_i \;\; \forall h, i$$

$$\boldsymbol{\beta}_i \geq \mathbf{0} \;\; \forall i$$

Johnson & Kuosmanen (2009) How Operational Conditions and Practices Affect Productive Performance? Efficient Semi-Parametric One-Stage Estimators, SSRN working paper. http://ssrn.com/abstract=1485733

Advantages of StoNEZD estimator of δ

- Joint one-stage estimation of *f* and **δ**
- No functional form assumptions for frontier *f*
- Stochastic noise term included
- Inputs **x** can correlate with **z**
- Estimator of **z** variables is consistent, asy. efficient, asy. normal, and converges at standard parametric rate

Monte Carlo simulations

Banker and Natarajan (2008, *OR*)

$$y_i = (x_i^3 - 12x_i^2 + 48x_i - 37) \cdot \exp(-0.2 \cdot z_i + v_i - u_i)$$

$\sigma_u = 0.15$; $\sigma_v = 0.04$; $n = 200$

Root Mean Squared Deviation (RMSD) (%) for δ at different levels of correlation (ρ) between x and z variables

Estimator	ρ=-0.8	ρ=-0.2	**ρ= 0**	ρ=+0.2	ρ=+0.8
SFA cubic 1-stage MLE	15.7	9.5	**9.3**	9.5	15.4
SFA translog 1-st. MLE	43.9	36.9	**32.8**	32.2	40.7
2-DEA w. OLS	83.0	16.3	**10.8**	11.4	36.5
StoNEZD	17.4	10.5	**10.3**	10.8	15.3

Conclusions

- Least squares formulation of DEA provides a useful platform for imposing further structure and parametric assumptions
- Intimate links between DEA, SFA, Afriat, Shephard,..
- Is our field mature enough to leave the old quarrels and prejudices behind and pursue together a unified approach to productive efficiency analysis?