Aalto University
School of Economics

Efficiency analysis as a principle-agent game

An application to the regulation of electricity distribution networks

Timo Kuosmanen

HECER lunch seminar
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Plan for the presentation

- Regulation of electricity distribution in Finland: some recent developments

- Game theoretic interpretation of efficiency analysis as a principle-agent game

Motivation & background

- In Finland, firms and households are free to purchase electricity from any producer or intermediator, but the distribution remains a local monopoly

- The Finnish Energy Market Authority (EMV) regulates the distribution to counter abuse of monopoly power

- The acceptable rate of return for the invested capital based on the capital asset pricing model (CAPM)

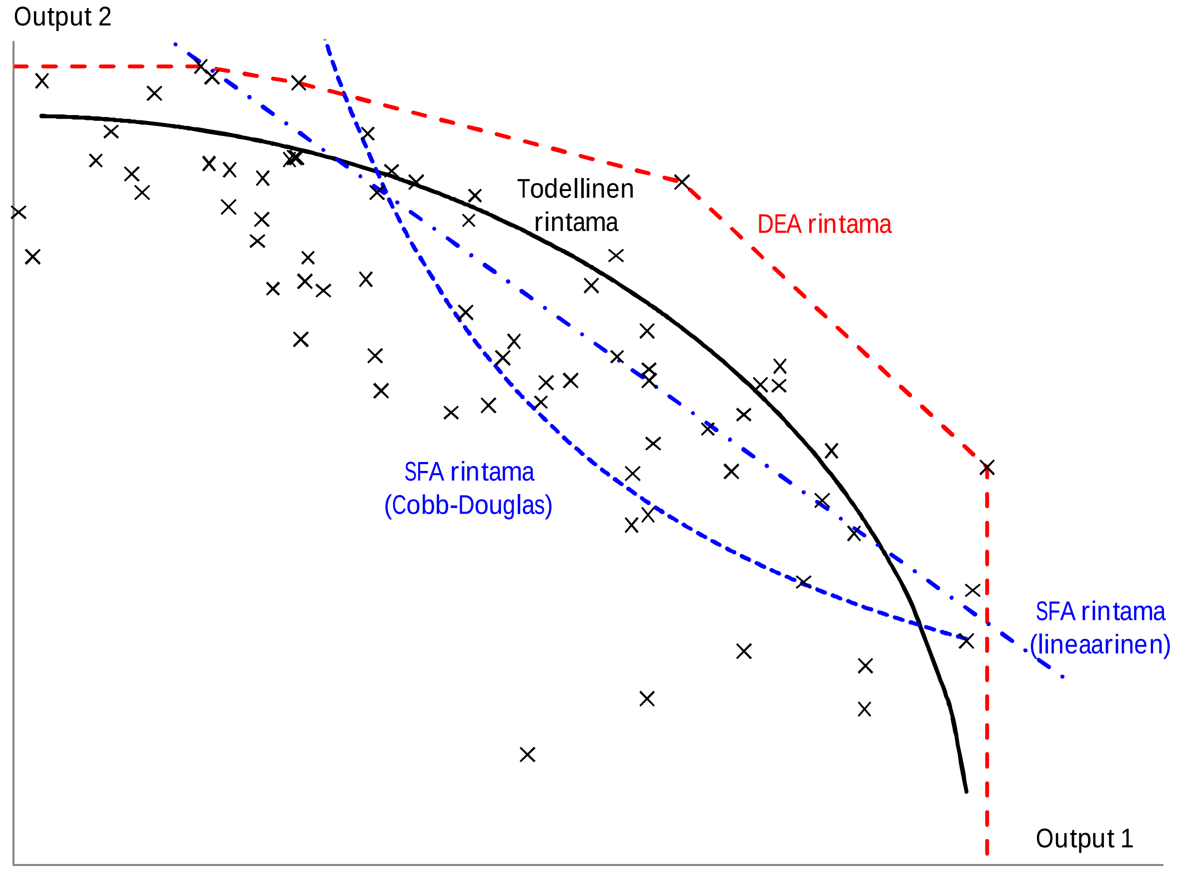
- The acceptable cost level based on the efficient cost frontier estimated by DEA and SFA methods

Motivation & background

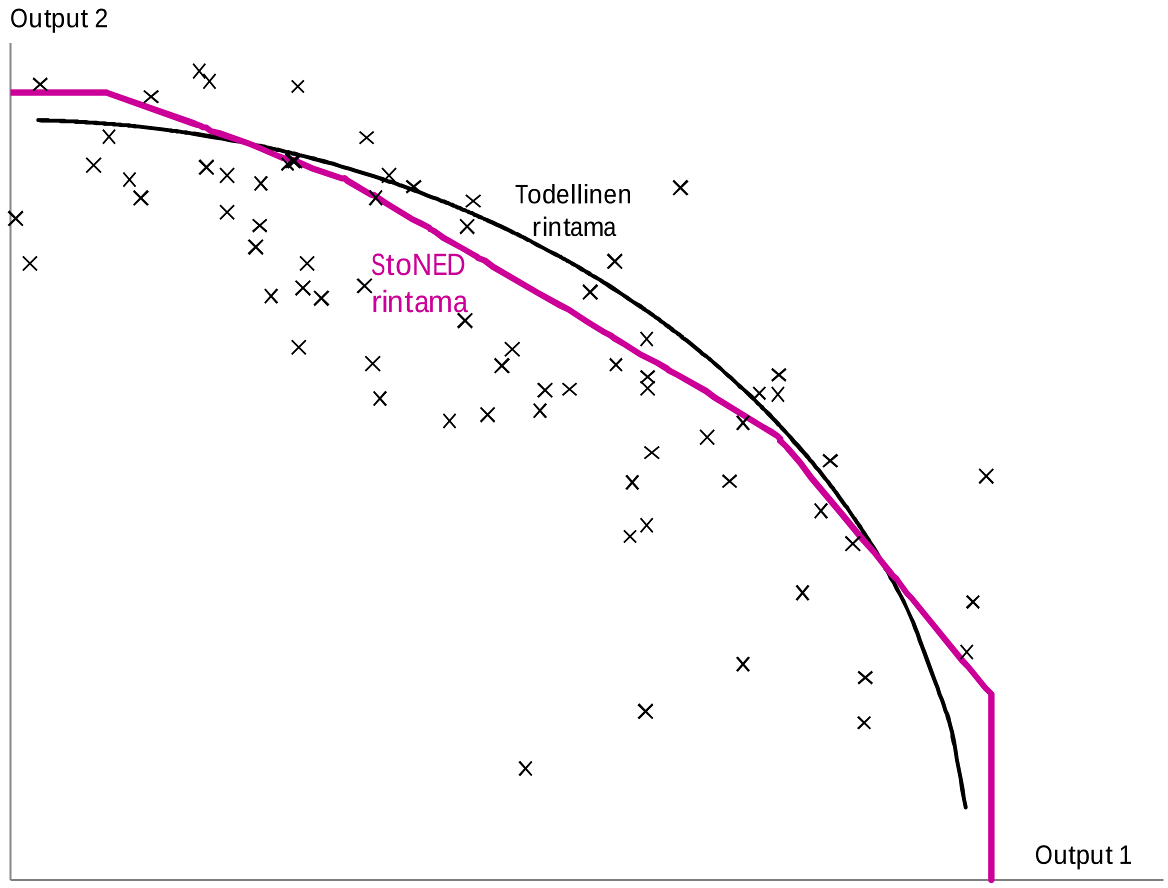
- Recent study by Kuosmanen, Kortelainen, Kultti, Pursiainen, Saastamoinen & Sipiläinen (2010) proposes several major reforms for the 3rd regulation period in 2012-2015: (www.energiamarkkinavirasto.fi)

- Replace the average of DEA and SFA efficiency by the StoNED estimator that combines a nonparametric frontier with stochastic inefficiency and noise terms

Simulated example with two outputs



Simulated example with two outputs



Cost frontier model

$$x = C(y_1, y_2, y_3) \cdot \exp(\delta z + u + v)$$

C is a convex, monotonic increasing cost function that exhibits CRS

y_1 is the weighted amount of energy transmission (GWh of 0.4 kV equivalents)

y_2 is the total length of the network (km)

y_3 is the total number of customers connected to the network

z is the proportion of underground cables

u is a half-normally distributed inefficiency term

v is a normally distributed noise term

StoNED estimator

Step 1: solve the CNLS problem

$$\min_{\gamma,\beta,\delta,\varepsilon} \sum_{i=1}^{n} \varepsilon_i^2$$

$$s.t.$$

$$\ln x_i = \ln \gamma_i + \delta z_i + \varepsilon_i \quad \forall i$$

$$\gamma_i = \beta_i' y_i \quad \forall i$$

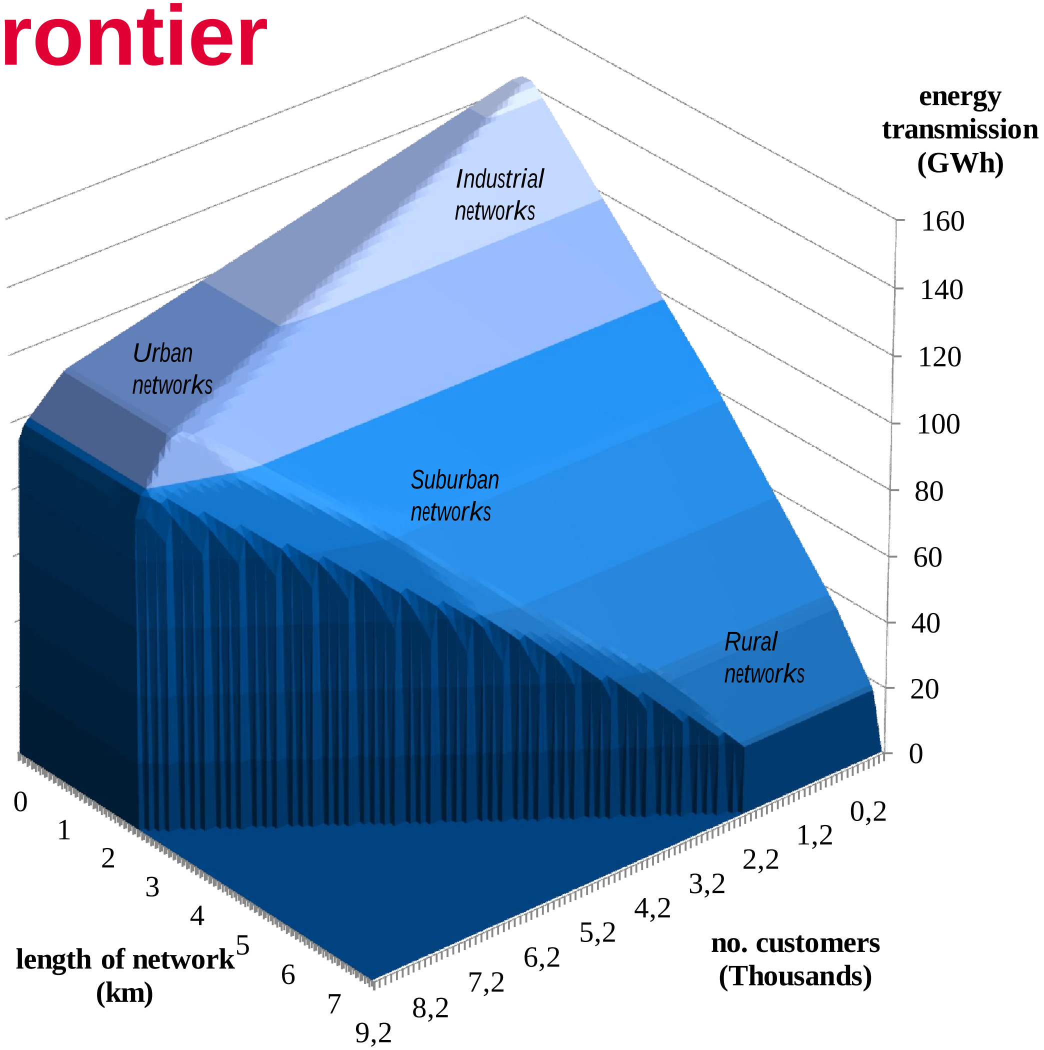
$$\gamma_i \geq \beta_h' y_i \quad \forall h, i$$

$$\beta_i \geq 0 \quad \forall i$$

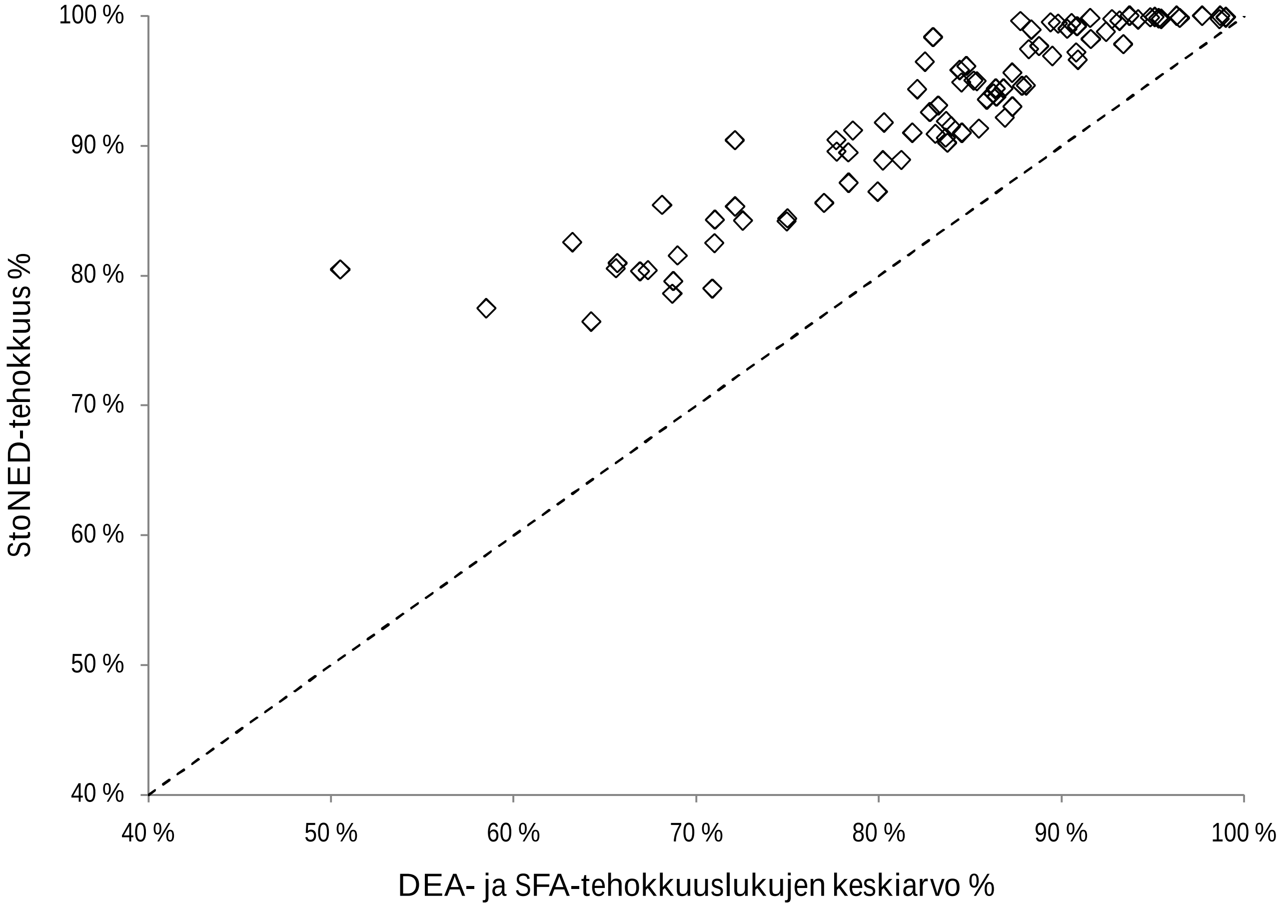
Step 2: Estimate the variance of *u* based on the skewness of residuals.

$$\hat{C}^{StoNED}(y) = \gamma_i + \hat{\sigma}_u \sqrt{2 / \pi}$$

StoNED frontier



Comparison of efficiency estimates



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Efficiency improvement targets in Mill. € (prices of 2008)

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DEA	141,382	1,589	2,223	0,000	27,654
SFA	95,481	1,073	3,888	0,024	12,830
ka*	118,431	1,331	2,959	0,016	20,242
StoNED	47,508	0,534	1,326	0,000	11,113

Game theoretic perspective

- DEA has a game theoretic interpretation as a Nash equilibrium of a principle-agent game (Banker 1980, *EJOR*)

- DEA is a restricted special case of StoNED (Kuosmanen & Johnson 2010, *Operations Research*)

Game theoretic perspective

- DEA has a game theoretic interpretation as a Nash equilibrium of a principle-agent game (Banker 1980, *EJOR*)

- DEA is a restricted special case of StoNED (Kuosmanen & Johnson 2010, *Operations Research*)

$\Rightarrow$ StoNED must have a game theoretic interpretation as well

Backward induction: how to specify an elegant/intuitive game that has the StoNED method as its Nash equilibrium?

Data Envelopment Analysis (DEA)

Outputs **y**, *Total cost x*

Envelopment problem

$$\min_{\boldsymbol{\lambda}\geq\mathbf{0}} \theta$$

$$s.t.$$

$$\theta x_i \geq \sum_{h=1}^{n} \lambda_h x_h$$

$$\mathbf{y}_i \leq \sum_{h=1}^{n} \lambda_h \mathbf{y}_h$$

Multiplier problem

$$\max_{\mathbf{p}} \frac{\mathbf{p}'\mathbf{y}_i}{x_i}$$

$$s.t.$$

$$\frac{\mathbf{p}'\mathbf{y}_h}{x_h} \leq 1 \quad \forall h = 1,\dots,n$$

Principle-agent game

- **Players**: principle (= regulator), agent (= regulated firm *i*)
- **Moves**: principle selects the benchmark firm *h* (*h*=1,…,*n*), agent selects output *k* (*k*=1,2,3). Simultaneous moves.
- **Strategies**: mixed strategies permitted
- **Payoffs**: baced on relative efficiency (zero-sum game)
  Agent: $(y_{ik} / x_i) / (y_{hk} / x_h)$
  Principle: $1 - [(y_{ik} / x_i) / (y_{hk} / x_h)]$

Data Envelopment Analysis (DEA)

Outputs **y**, *Total cost x*

Minimax formulation:

$$\min_{h \in \{1,\dots,n\}} \max_{\mathbf{p}} \left(\frac{\mathbf{p}'\mathbf{y}_i}{x_i} \right) \Big/ \left(\frac{\mathbf{p}'\mathbf{y}_h}{x_h} \right)$$

The optimal solution to the DEA problem provides the mixed strategy Nash equilibrium of the previous principle-agent game

Interpretation

- In the DEA game, principle plays a separate game with each agent (lack of coordination)

- Principle chooses the benchmark to minimize efficiency (advocator of energy consumers)

- In the regulatory model of EMV, the role of EMV is to balance the interests of consumers and energy companies (and avoid interruptions)

Variation 1

- **Sequential moves**: principle moves first, agent observes the benchmark and selects the output

$\Rightarrow$ Non-convex FDH efficiency is the Nash equilibrium

- Agent better off if the benchmark is fixed first
- Principle better off in the simultaneous move game

Variation 2

Union of agents:

- What if agents form a union to represent them and play a single coordinated game with the principle?

- **Payoffs**: baced on relative efficiency

  Union: $\Sigma_i (y_{ik}/ x_i) / (y_{hk} / x_h)$

  Principle: $1 – [\Sigma_i (y_{ik} / x_i) / (y_{hk} / x_h)]$

- Nash equilibrium: $$\min_{h \in \{1,\ldots,n\}} \max_{\mathbf{p}} \sum_{i=1}^{n} \left(\frac{\mathbf{p}'\mathbf{y}_i}{x_i} \right) \Big/ \left(\frac{\mathbf{p}'\mathbf{y}_h}{x_h} \right)$$

Variation 2

Union of agents:

- The parametric programming (PP) model by Aigner and Chu (1968, *AER*) gives the Nash equilibrium

- The PP frontier envelops the DEA frontier
- Principle is better off in the case of a union, none of the agents is better off

$\Rightarrow$ Agents do not have incentive to let a union choose the same strategy for all agents

$\Rightarrow$ If the union selects firm-specific output separately for each agent, we fall back to the DEA case

Nash equilibrium of a bi-matrix game

- Mangasarian and Stone (1964) show that the Nash equilibrium of a two-person, nonzero-sum game with a finite number of pure strategies is equivalent to the optimal solution to a certain quadratic programming problem with linear constraints.

- CNLS estimator is obtained as the optimal solution to a quadratic programming problem.

- To develop the game theoretic interpretation, we can specify the payoff matrices to match the CNLS problem

Variation 3

Benevolent regulator:

- Suppose the objective of the principle is to minimize the sample variance of efficiency scores (show the industry in positive light)

- Payoff of the principle:

$$1-\sum_{i=1}^{n}\left(Eff_{i,h}-\overline{Eff}\right)^2, \quad Eff_{i,h}=\left(\frac{\mathbf{p}'\mathbf{y}_i}{x_i}\right)\Big/\left(\frac{\mathbf{p}'\mathbf{y}_h}{x_h}\right)$$

Thank you!

Ideas, comments, feedback & help are more than welcome