WHAT IS THE ECONOMIC MEANING OF FDH?

A Reply to Thrall

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1. Introduction

In a recent issue of the *Journal of Productivity Analysis*, Thrall (1999) called for abandoning the Free Disposable Hull (FDH, Deprins et al., 1984) approximation of production possibilities as economically meaningless in comparison to the Convex Monotone Hull (CMH; Banker et al., 1984) approximation. This strong conclusion was solely based on Thrall's Principal Theorem, which essentially demonstrates that FDH can give a technically efficient classification to output-input vectors that are inefficient in terms of profit maximisation, i.e. at all non-negative price vectors there exists an alternative output-input vector that yields higher profit. In this short communication, we argue that the economic meaning of the competing empirical production sets cannot be inferred from this theorem. Specifically, we demonstrate that both empirical production sets are economically equally meaningful under the economic conditions that underlie Thrall's theorem. In addition, we demonstrate that FDH can be economically more meaningful than CMH under non-trivial alternative economic conditions.

2. Technical efficiency versus profit efficiency

FDH relies on the sole assumption that production possibilities satisfy free disposability. The key difference between FDH and CMH is that CMH imposes the additional assumption that production possibilities are convex[1]. Free disposability is a generally accepted assumption in production economics. Färe and Grosskopf (1983) show that imposing free disposability in fact implies a congestion-adjusted approximation. Note that this congestion adjustment is harmless as far as the usual (Pareto-Koopmans or Debreu-Farrell) notions of technical efficiency are concerned. By contrast, to the best of our knowledge, there appear to be no valid theoretical arguments for assuming a priori that production possibilities are truly convex (see also McFadden, 1978, pp. 8-10). In addition, various empirical studies of non-trivial industries suggest violations of the convexity hypothesis (see e.g. Hasenkamp, 1976; Kuosmanen, 1999; Dekker and Post, 1999). Therefore, FDH seems to have a comparative advantage for analysing technical efficiency.

Ironically, Thrall's negative conclusion towards FDH originates from confusing technical and economic efficiency criteria. Thrall considered the evaluation of efficiency in terms of profit maximisation at given prices, but he used the Pareto-Koopmans notion of technical efficiency as the only efficiency criterion. Theoretically, technical efficiency is a *necessary* but generally not a *sufficient* condition for overall economic efficiency (Farrell, 1957). Therefore, FDH technical efficiency cannot guarantee profit maximisation, as Thrall correctly points out, but it should be added that CMH technical efficiency does not guarantee it either.

When an appropriate efficiency criterion that complies with the assumed profit maximisation objective is adopted (i.e. *profit efficiency*, as originally proposed by Nerlove, 1965)[2], both approximations appear equally meaningful. The Nerlovian profit efficiency criterion compares the obtained profit level to the highest profit level attainable given technology and output and input prices: an output-input combination is labelled efficient if, for given prices, no higher profit level is shown to be attainable. A standard result in production analysis is that production sets can be 'monotonised' and 'convexified' without harm for the purpose of measuring profit efficiency, because monotonisation and convexification do not interfere with the maximum profit level (see e.g. Varian, 1984). In fact, from production economics we know that cost, revenue and profit functions associated with a particular technology equal those corresponding to its appropriately convexified counterpart, a point which is nicely illustrated by Diewert (1982; p. 538-540). We conclude that maximum profit levels associated with the FDH and CMH approximations are identical. Therefore, we do not see any reason to discriminate between the two approximations as regards economic efficiency**.**

3. Endogenous prices and price uncertainty

The equivalence result mentioned at the end of the previous section only applies to Nerlovian profit efficiency in Farrell's framework, which is economically relevant if producers take exogenously fixed prices as given. That is a valid assumption under economic conditions of perfect competition or price rationing under perfect certainty. In real-life industries, however, these conditions are mostly not satisfied even by approximation[3]. In many cases, prices and quantities of inputs and outputs are mutually dependent and the price-taking assumption is no longer valid. Also, decision-makers frequently face uncertainty about ex post prices when ex ante allocating resources and producing output (McCall, 1967; Sandmo, 1971). In these instances, the economic justification of convexification breaks down, and FDH generally becomes economically more meaningful than CMH.

First, if input and output prices vary with quantities demanded or supplied, the evaluated firm can in general not be compared with a convex combination of firms with a different output-input structure, which are confronted with different output and input prices. Indeed, firms of which convex combinations dominate the evaluated firm may actually be associated with strictly *lower* profit levels. This point can be illustrated using the example considered by Thrall (1999). That example involved three firms producing two outputs, y_1 and y_2, from a single input x. The observed vectors (y_1, y_2, x) are (1, 10, 1), (2,2,1) and (10,1, 1). The output-input vector (2,2,1) is FDH efficient, as it is not dominated by any of the two other observations, but CMH inefficient, because convex combinations of (1, 10, 1) and (10,1, 1) do dominate it. However, this does not necessarily imply that (2,2,1) is not profit-maximising. Consider, for example, the following linear inverse demand functions for the two outputs (with p_i the price of y_i (i=1,2)): $p_1 = 10 - y_1 + ½ y_2$; $p_2 = 10 - y_2 + ½ y_1$. The output bundle (2,2) is associated with revenue of 36, while output bundles (1,10) and (10,1) both yield revenue of 19 only. Since all three firms use the same input amount, the profit associated with (2,2,1) is necessarily higher than that corresponding to (1,10,1) and (10,1,1). Hence, the output-input vector (2,2,1) cannot be demonstrated to be economically inefficient. While the argument for convexification breaks down if prices are endogenous, free disposability can be maintained without harm in the economically meaningful region where marginal costs of inputs and marginal revenues of outputs are nonnegative.

Second, convexification requires that producers have perfect information about input and output prices when fixing their production plans. When prices are uncertain, however, it can be unfair and misleading to measure economic efficiency *at the ex post prices.* Rather, profit realisations at all possible ex post price scenarios have to be taken into account. FDH is economically appealing under uncertainty due to the fact that for every interior point there exists an actually observed point that yields higher profit at *all* possible (non-negative) price vectors. This has a natural interpretation in terms of *first-order stochastic dominance*, a well-known decision criterion for choice under uncertainty. In particular, under FDH the profit distribution of an interior point is always stochastically dominated by that of an actually observed point (see Kuosmanen and Post, 1999, for a more elaborate discussion). This needs not be the case with CMH. Although for each interior point there exists at *every single* price vector an observed DMU that yields higher profit, CMH reference units need not yield higher profits at *all* price vectors. Hence, CMH does not have the same stochastic dominance interpretation as FDH.

4. The interpretation of substitution rates

A final point concerns Thrall’s claim that CMH efficiency analysis provides the researcher with additional information on the terms of substitution. We do not believe this provides a valid argument in favour of CMH and against FDH. Firstly, one has to distinguish between the (supply-side) technical substitution properties on the one hand and the (demand-side) preference-related substitution effects on the other. In contrast to Thrall’s claims, CMH can only approximate the former type of substitution rates, and only so for a truly convex production set. Secondly, although FDH does not directly provide estimates of the technical substitution properties, it may be complemented by various nonparametric or parametric approaches (see Kuosmanen (1999) for a detailed discussion and references). For example, Thiry and Tulkens (1992) used a two-stage technique that first filters out FDH inefficient DMUs and subsequently estimates the frontier using parametric regression techniques.

5. Conclusions

We pointed out that FDH remains an economically meaningful empirical production set. Specifically, we showed that Thrall’s main argument against FDH, concerning the potential disparity between technical and profit efficiency classifications, builds on an inappropriate efficiency criterion. We subsequently demonstrated that FDH and CMH approximations actually lead to exactly the same profit efficiency results when a suitable criterion is applied. In addition, we discussed non-trivial economic conditions (imperfect competition and price uncertainty) under which FDH may even become economically more meaningful than CMH. We want to underline that there are no a priori economic grounds to believe that the true production set is convex. Therefore, FDH has a comparative advantage over CMH as regards technical efficiency analysis and the related decomposition of economic efficiency into allocative and technical components.

Nevertheless, some non-economic arguments in favour of CMH can be acknowledged. Firstly, simple linear programming techniques can compute e.g. Farrell (1957) technical efficiency measures relative to CMH (see Banker et al., 1984). However, we would add that FDH Farrell estimates are equally simply obtained from integer programming, which is no problem to modern-day solvers, or from enumeration, which does not require mathematical programming at all (see Tulkens, 1993). Second, if the technology would be truly convex - by some strike of luck - the CMH approximation may yield better technical efficiency estimates than the FDH approximation. This, however, would in principle require some empirical verification of the convexity property, which to the best of our knowledge is not easily obtained. Moreover, it is possible to reduce the small sample error associated with FDH using information from the asymptotic distributions of efficiency estimates (Park et al., 1997), or alternatively from simulated empirical distributions generated with bootstrapping techniques (e.g. Simar and Wilson, 1998). Including additional local production information (see e.g. Thannassoulis and Allen, 1998) could also help to remedy this problem. Finally, if the production set would indeed happen to be convex, possible advantages of CMH over FDH generally vanish for large samples.

Notes

1. Free disposability and convexity are assumptions about the true production possibilities. Both FDH and BCC impose the additional (determinism) assumption that observed input-output vectors are a sample from the true production possibilities.
2. See also Banker and Maindiratta (1988) and Chambers et al. (1998).
3. Nerlove (1965, pp. 90) already criticised Farrell's (1957) treatment, which essentially remains embedded in modern DEA, for the fact that it is not generally applicable under imperfect competition.

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