NON-PARAMETRIC PRODUCTION ANALYSIS IN NON-COMPETITIVE ENVIRONMENTS

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ABSTRACT We extend the non-parametric methodology for empirical production analysis to deal with *endogenous* prices. As price endogeneity is often complemented by price uncertainty, we consider both the case of certain prices and the case of uncertain prices. The extensions are fully compatible with existing tools for eliciting and representing technology and price information, and preserve**s** the tractable mathematical programming structure of the original methodology. An empirical application to the Dutch electricity distribution sector illustrates our extension.

***Key words**: non-parametric production analysis, Data Envelopment Analysis, endogenous prices, electricity distribution sector.*

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1. INTRODUCTION

A systematic methodology for empirical production analysis originated from the work by Afriat [1], Hanoch and Rothschild [2], Diewert and Parkan [3] and Varian [4], building on the activity analysis of Koopmans [5], the duality theory of Shephard [6], the efficiency analysis of Farrell [7], and the revealed preference approach of Samuelson [8]. In the context of firm-specific efficiency analysis à la Farrell, the nonparametric approach is also known as Data Envelopment Analysis (DEA); due to Charnes *et al.* [9], Banker *et al.* [10], Charnes *et al.* [11]. The distinguishing feature of the methodology is its non-parametric nature; the methodology does not use assumptions about the functional form of the production relationships and the statistical distribution of the inefficiencies. This feature makes it possible for the data to 'speak for themselves' rather than being forced to use the idiom of some imposed functional form and distribution function. This is an attractive feature because production theory does not forward specific functional forms, and reliable empirical specification tests are not available in many cases. Further, already Debreu [12] and Farrell [7] expressed their concern about the ability to measure prices accurately enough to make good use of economic efficiency measurement. For example, accounting data can give a poor approximation for economic prices (i.e. marginal opportunity costs), because of debatable valuation and depreciation schemes. The non-parametric methodology includes a variety of powerful tools for dealing with incomplete price information (most notably the use of polyhedral price sets). Finally, the methodology is computationally attractive, as it generally requires straightforward Linear Programming only.

Despite these attractive features, the current non-parametric methodology builds on a simplified theoretical model of the firm. Specifically, (following the neoclassical theory of the firm) the methodology assumes that firms (or decision-making units) take prices as exogenously given. Frequently, this assumption is too stringent to give a reasonable approximation of firm-behavior. In many industries, firms have market power and the production plans of individual firms affect the market prices. For example, the current market conditions in the Dutch electricity sector suggest that electricity distributors in the Netherlands can affect electricity prices in a non-trivial way (see Section 5). In fact, market power frequently provides the motivation for investigating efficiency in the first place, as firms with market power can operate inefficiently without being pushed out of business by (efficient) competitors. This applies for private firms that operate in an industry with entry barriers, and it also applies for public organizations (like utilities) that operate in a non-competitive regulated environment.

There is a wealth of theoretical models to explain firm behavior under conditions of imperfect competition (see e.g. Hart [13] for an overview). However, the empirical implementation of these theories is extremely difficult, because the information requirement is enormous. For example, one generally needs detailed assumptions about the market structure (both demand and supply side) to choose from different theoretical models. Also, since the appropriate model critically depends on the idiosyncrasies of the industry under evaluation, it is practically impossible to find a model that is general enough to apply to a wide variety of industries. Finally, in many cases the computational burden associated with detailed theoretical models practically excludes empirical application. These complications at least partly explain why empirical research has not 'caught up' with the theory.

Varian ([4], Section 10) brought to the attention the need to extend the non-parametric methodology to account for imperfect competition. Following Varian's suggestion, we provide a general framework for dealing with endogenous prices.[1] Frequently, price endogeneity is complemented by price uncertainty. For example, if prices are endogenous, a particular production plan can be associated with different price equilibria (compare with Debreu [14]; see also Grodal [15]), which immediately implies ex ante price uncertainty. In

[1] In fact, our general framework includes as special cases the tools that Varian [4] proposed to deal with price endogeneity.

addition, firm owners usually imperfectly observe the interaction between firm actions on the one hand and market prices on the other, which introduces further price uncertainty. For these reasons, we integrate our approach with the approach to uncertainty proposed by Kuosmanen and Post [16].

The practical limitations faced in empirical research motivate us to develop a framework that (1) does not require detailed information about the market structure, (2) is compatible with the existing tools for representing information on technology and prices, and (3) is computationally tractable. We do realize that such a general approach necessarily comes at the cost of a loss of power. Therefore, we will explicitly discuss various ways of improving power within our framework. In addition, we strongly believe that this ' non-parametric' approach forms a good starting point, even if it involves only little power. The fact is that violations of our general efficiency conditions provide strong evidence against optimizing behavior. After all, if observations are inconsistent with general conditions, then there is little hope that they will pass more specific tests that builds on more detailed assumptions.

In the following, we focus on the subject of efficiency analysis, i.e. testing whether or not observed production data are consistent with optimizing behavior, and quantifying inefficiencies or the degree of violations of optimizing behavior. As discussed in Varian [4], the non-parametric methodology can also be used for (1) testing hypotheses about the production technology or the prices, (2) recovering the technology or prices, and (3) forecasting economic behavior based on observed past behavior. For the sake of compactness, we abstract from these alternative applications of the non-parametric approach. However, our generalized efficiency measures can be extended in a relatively straightforward way to generalize the complete non-parametric framework, e.g. along similar lines as those followed by Varian [4].

The remainder of this paper is organized as follows. In Section 2, we review the traditional non-parametric methodology, with special attention for the case where only limited technology and price information is available. In Section 3**,** we extend the original methodology to account for endogenous prices. Section 4 considers price uncertainty in addition to price endogeneity. Section 5 discusses how to elicit and represent production and price information so as to increase the power of the proposed analysis tools. Section 6 includes an empirical application to the Dutch electricity market. Finally, Section 7 summarizes our conclusions and gives an agenda for further research.

2. THE TRADITIONAL APPROACH

Our purpose is to evaluate the economic optimizing behavior of firms indexed by $j \in J \equiv \{1,...,n\}$ that face a (non-empty and closed) production possibilities set $T \in \mathbf{R}^q$ involving q netputs (positive netputs represent outputs, negative netputs represent inputs).[2] We are essentially concerned with testing whether observed firm behavior is *efficient,* i.e. consistent with optimal behavior. Throughout the text, the observed netputs of the firms are represented by the matrix $Y \equiv (y_1 ... y_n)$ or the discrete set $S \equiv \{y_1, \cdots, y_n\}$, with $y_j \equiv (y_{1j} ... y_{qj})^T \in T$.

Different firms have different production objectives. To assess firm efficiency, it is important to apply efficiency criteria that correctly reflect the economic objectives of the firms under evaluation. For example, the finding that the firm could substantially increase a particular netput without worsening any of the other netputs is not economically relevant for the firm if the price of that netput is negligible or if the demand for that netput is determined exogenously (as for utilities that operate in a service area that is fixed by regulation). In this paper, we focus on Nerlove' s [17] concept of *profit efficiency*, which builds on the traditional

[2] Throughout the text, we will use $\mathbf{R}^m$ for an *m*-dimensional Euclidean space, $\mathbf{R}^m_+$ and denotes the positive orthant.

theory of the competitive firm that seeks to maximize profit given the production technology and the input-output prices. In many empirical research situations, firms face problems that are more complex than unconstrained profit maximization. In practice, we typically need to impose additional restrictions e.g. due to the non-discretionary nature of exogenously fixed netputs. Moreover, the firm may face additional cost or revenue constraints; e.g. the cost of some inputs may not exceed a certain a priori budgeted sum. Still, enriching efficiency analysis with those types of additional constraints is relatively straightforward, and operational solutions are well documented elsewhere (e.g. Färe and Primont [18]; Färe and Grosskopf [19]; and Grosskopf et al. [20]). Therefore, we choose to focus on the basic case of profit maximization in the following.

The conventional methodology assumes that the firm faces an exogenously given (firm-specific) price vector $p \equiv (p_1 \cdots p_q) \in \mathbf{R}_+^q$. The prices effectively reflect exogenous market conditions. Different firms can face different market conditions, and comparing the profit of firms with different conditions is not fair to firms that face unfavorable conditions. Therefore, the tests below compare the profit of the evaluated firm with the hypothetical profits of other firms at the prices of the evaluated firm. However, for notational convenience, we will suppress firm-specific indexes.

We call a firm $j \in J$ *profit efficient* if and only if

$$\pi(y_j, p, T) \equiv \max_{y \in T} \{p(y - y_j)\} = 0 . \quad (1)$$

In many cases, the 'profit shortfall' $\pi(y_j, p, T)$ cannot be computed directly, as full information about technology and prices is not available. This problem is typically remedied by using an inner bound approximation for the production set and an outer bound approximation for the price set.

The inner bound for the production set is typically the discrete set of observations *S*, or alternatively an empirical production set constructed from *S*. A prime example of such an empirical production set is the convex monotone hull of the observations:

$$\text{com}(S) \equiv \{ y \in \mathbf{R}^q : y \leq \lambda^T Y; \lambda^T e = 1; \lambda \in \mathbf{R}^n_+ \}, \tag{2}$$

with $e \equiv (1 \ldots 1)^T$ for a unity vector of dimension conforming to the rules of matrix algebra.

This polytope has become the 'standard' in the non-parametric literature; it was used in e.g. Banker *et al.* [10], Varian [4], Charnes *et al.* [11], and Banker and Maindiratta [21]. One particularly appealing aspect of this set is that monotonicity and convexity are 'harmless' regularity properties in the context of measuring profit efficiency. Specifically, monotonicity and convexity do not interfere with the test results, i.e. $\pi(y_j, p_j, \text{com}(S)) = \pi(y_j, p_j, S)$. In fact, a basic insight within the efficiency analysis literature is that a netput vector situated within the interior of the convex monotone hull of a production set will always be classified as profit inefficient, independent of the price vector that is used (see also the example below).

Apart from monotonicity and convexity, additional technological assumptions can be imposed (or tested). For example, using the conical convex monotone hull rather than the convex monotone hull (as in Charnes *et al.* [9]) incorporates the assumption of constant returns-to-scale. We do not provide a detailed discussion in this paper but refer to Färe *et al.* [22] for an overview of a variety of alternative empirical production sets used in the non-parametric methodology. Also, we do not express a preference for a particular empirical set. Rather, for the sake of generality, we will phrase in terms of a general empirical production set denoted by $\Theta(S)$.

As for incorporating price information, the standard approach is to restrict the unknown price vector to a specific domain. The price domain is usually represented in the form of polyhedra, as in the so-called assurance region (AR) or weight-restricted (WR) models (first introduced by Thompson *et al.* [23], [24]; see Allen *et al.* [25], for a survey).[3] Throughout this paper, we will use the following polyhedron:

$$\Pi(A,b) \equiv \left\{ \rho \in \mathbf{R}_{+}^{q} : \rho A \geq b \right\}. \tag{3}$$

This polyhedron contains all non-negative price vectors that satisfy l linear inequalities characterized by the $l \times q$ matrix A and the $l \times 1$ vector b. Note that the price domain contains $\mathbf{R}_{+}^{q}$ as its most general special case (i.e. if A and b are void). Also, the matrix A and vector b can be specified to restrict the prices (or price ratios) to unique values.

The following theorem (with proof omitted as obvious) demonstrates how the inner technology bound and the outer price bound can yield an operational efficiency test:

THEOREM 1 *If* $\Theta(S) \subseteq T$ *and* $p \in \Pi(A,b)$, *then* $\pi(y_j, \rho, T)$ *is bounded from below by*

$$\theta(y_j, \Pi(A,b), \Theta(S)) \equiv \min_{\rho \in \Pi(A,b)} \left\{ \pi(y_j, \rho, \Theta(S)) \right\} = \min_{\rho \in \Pi(A,b)} \left\{ \max_{y \in S} \left\{ \rho(y - y_j) \right\} \right\}$$ and

$\theta(y_j, \Pi(A,b), \Theta(S)) = 0$ *is a necessary condition for* $\pi(y_j, \rho, T) = 0$.

As discussed e.g. by Kuosmanen and Post [16], the measure $\theta(y_j, \Pi, \Theta(S))$ can be expressed in equivalent dual form as a multidimensional distance measure relative to a

[3] AR-WR models were originally developed to include information about the marginal properties or shadow prices of the production set for measuring technical efficiency. As demonstrated by Kuosmanen and Post [39], the AR-WR approach can also be adopted for representing price information for measuring economic efficiency. Interestingly, in the traditional model of profit maximization, the optimal shadow prices equal the true prices, and hence both approaches effectively are equivalent. However, this equivalence breaks down in case of imperfect competition.

polyhedral reference set constructed from the convex hull of the empirical production set $\Theta(S)$ and the negative polar of the price polyhedron $\Pi(A,b)$. To simplify the discussion, we will abstract from dual formulations in this paper and focus on the primal perspective exclusively.

Figure 1 illustrates the above efficiency analysis tools. It displays the netputs of 4 firms (firm 1, 2, 3, and 4; netput values in brackets) facing a two-output technology. We assume that technology information is restricted to the discrete set of netputs of these firms, i.e. $\Theta(S)=S=\left\{\begin{pmatrix}2\\10\end{pmatrix},\begin{pmatrix}3\\7\end{pmatrix},\begin{pmatrix}5\\3\end{pmatrix},\begin{pmatrix}10\\2\end{pmatrix}\right\}$. Price information is restricted to the information that the price of the first netput is at least half and at most twice the price of the second netput.[4] In addition, we use a normalization constraint to impose the restriction that the prices should sum to one. This price information can be represented by $A=\begin{bmatrix}2 & -1\\-1 & 2\\1 & 1\\-1 & -1\end{bmatrix}$ and $b=\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$.

Under these technology and price assumptions firm 2 and firm 3 are classified as profit inefficient: at each price vector within $\Pi(A,b)$ either firm 1 or firm 4 achieves a higher profit level. Figure 1 graphically illustrates this result. The shaded area (a polyhedral reference set that can be constructed from the convex monotone hull of S plus the negative polar of $\Pi(A,b)$) contains all netput vectors that are inefficient at all possible price vectors $\rho\in\Pi(A,b)$. Both firm 2 and firm 3 lie within the interior of that set, and are thus classified as inefficient. Putting it somewhat differently, no possible price vector $\rho\in\Pi(A,b)$ can 'rationalize' observed behavior of firm 2 and firm 3.

[4] For transparency, different firms face the same price conditions in our numerical examples.

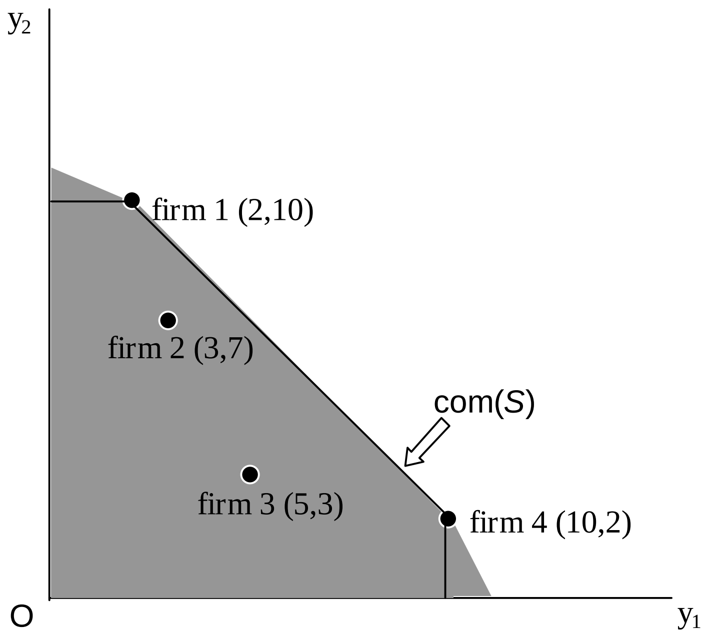

Fig. 1 Polyhedral reference set for traditional profit efficiency. Firm 2 and firm 3 are classified as inefficient; at all possible prices, these firms involve less profit than either firm 1 or firm 2 (or both).

3. ENDOGENOUS PRICES

To generalize the traditional approach, we consider the classic Cournot [26] setting where netput quantities are the decision variables, while prices are set on the market.[5] That is, in contrast to the standard nonparametric approach, we assume that prices (partly) depend on the netputs of the firm. The relation between prices and quantities is represented by the general inverse demand function (IDF) $\Phi(y): T \rightarrow D$. The IDF maps the netputs of the evaluated firm onto the price domain $D \subseteq \mathbf{R}_{+}^{q}$. Note that only the netputs of the evaluated firm enter in the IDF. However, this does not mean that prices cannot depend on the netput choices of the peers (which may compete with the evaluated firm in the same input and output markets). The IDF effectively captures the reactions to the netput decision of the evaluated firm by the other firms, as well as that by other competitors in input and output markets, output consumers and input suppliers. As in the traditional approach, we allow for a firm-specific IDF, so as to allow for differences in the netput demand conditions and the economic microenvironment. The IDF effectively reflects exogenous market conditions. As before, different firms can face different market conditions, and therefore the tests below compare the profit of the evaluated firm with the hypothetical profits of other firms at the IDF of the evaluated firm.

For simplicity, we adhere to the conventional profit maximization objective of the firm (see Section 5 for discussion), and adopt the most straightforward modification of the profit efficiency notion. Specifically, under endogenous prices a firm is $j \in J$ *profit efficient* if and only if

$$\pi^E(y_j, \Phi, T) \equiv \max_{y \in T} \left\{ \Phi(y)y - \Phi(y_j)y_j \right\} = 0 . \tag{4}$$

As in the traditional case, this measure can not be computed without full technology information. As before, we can approximate T by an empirical production set $\Theta(S)$. Further, the specification of the IDF requires detailed information about the structure of demand and supply forces. We think a requirement for such detailed information is not consistent with the non-parametric orientation. In addition, detailed structure invariantly reduces the generality of the framework. Finally, the computational burden associated with detailed structure may exclude practical application.

In line with the existing use of bounding price sets, we propose to specify bounds on the price function at different ranges of netput values. If the demand and supply conditions faced by the firm do not depend too heavily on the netput quantities, we can safely use a generic price domain $\Pi(A,b) \supseteq D$. However, if price endogeneity is a more serious problem, i.e. demand and supply do vary substantially with the netput quantities, then the 'global' domain

[5] Note that our approach covers a more general class of non-competitive settings than the orginal Cournot [26] duopoly: it fits to all situations where the firm sets the quantity and the demand sets the

D (including *all* possible price realizations) is likely to be very large; in some cases it can cover almost the entire positive orthant. Evidently, using a generic price domain like $\Pi(A,b)$ would involve little power. In such cases, the use of tighter boundaries by exploiting 'local' price information is therefore recommendable.

To incorporate local price information in the analysis, we partition the netput space T into appropriate subsets t_i, indexed by $i \in M \equiv \{1,\cdots,m\}$, such that $T = \bigcup_{i \in M} t_i$. For each subset $i \in M$, we specify a 'local' polyhedron

$$\Pi(A_i, b_i) \equiv \{\rho \in \mathbf{R}_+^q : \rho A_i \geq b_i\} \tag{5}$$

such that $\Phi(y) \in \Pi(A_i, b_i) \quad \forall y \in t_i$. Since the subsets t_i may overlap, we will use the *price correspondence*

$$\Omega(y) \equiv \bigcap_{i \in M : y \in t_i} \Pi(A_i, b_i) = \{\rho \in \mathbf{R}_+^q : \rho A_i \geq b_i \quad \forall i \in M : y \in t_i\}. \tag{6}$$

Using these local polyhedra, we can generalize Theorem 1:

THEOREM 2 *If* $\Theta(S) \subseteq T$ *and* $\Phi(y) \in \Omega(y) \quad \forall y \in T$, *then* $\pi^E(y_j, \Phi, T)$ *is bounded from below by* $\xi(y_j, \Omega, \Theta(S)) \equiv \max_{y \in \Theta(S)} \left\{ \min_{\rho \in \Omega(y)} \{\rho y\} \right\} - \max_{\rho \in \Omega(y_j)} \{\rho y_j\}$ *and hence* $\xi(y_j, \Omega, \Theta(S)) \leq 0$ *is a necessary condition for* $\pi^E(y_j, \Phi, T) = 0$.

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price; such situations are often referred to as 'Cournot settings' in the economic literature.

PROOF $\Theta(S)\subseteq T$ and $\Phi(y)\in\Omega(y)\ \ \forall y\in T$ imply $\max_{y\in\Theta(S)}\left\{\min_{\rho\in\Omega(y)}\{\rho y\}\right\}\leq\max_{y\in T}\{\Phi(y)y\}$ and $\max_{\rho\in\Omega(y_j)}\{\rho y_j\}\geq\Phi(y_j)y_j$, and hence $\xi(y_j,\Omega,\Theta(S))\leq\pi(y_j,\Phi,T)$. ■

Note that the efficiency statistic $\xi(y_j,\Omega,\Theta(S))$ differs from the traditional statistic $\theta(y_j,\Pi(A,b),\Theta(S))$ in a subtle way. First, the ' minimax' structure is replaced by a ' maximin' structure: rather than evaluating the profit of the peers at the prices that are most favorable for the evaluated firm, each peer is evaluated at the prices that are least favorable for that peer. Further, the efficiency statistic generally evaluates the evaluated firm at different prices than the peers, so as to account for the possibility that different netput choices involve different prices. By contrast, the traditional framework assumes exogenous prices, and evaluates the evaluated firm and the peers at uniform prices.[6]

To illustrate the difference between our approach and the standard approach we recapture our example of Section 2, but now accounting for the possibility of endogenous prices. To enhance comparison with the original approach, we consider the generic price domain that was also used in Section 2, i.e. $\Omega(y)=\Pi(A,b)\quad\forall y\in T$. The ' maximum minimum profit' $\max_{y\in S}\left\{\min_{\rho\in\Omega(y)}\{\rho y\}\right\}$ is achieved by firm 1 and firm 4, which achieve at all possible prices a profit of at least $\frac{14}{3}$. Firm 3 remains profit inefficient, because it achieves at all possible prices a profit of at most $\frac{13}{3}<\frac{14}{3}$. By contrast, the available production and price information does not suffice to classify firm 2 as inefficient; the maximum profit of firm 2 is $\frac{17}{3}>\frac{14}{3}$. The gray area in Figure 2 represents all netput vectors that are classified as inefficient, i.e. that

[6] It is interesting to contrast these findings with those obtained by Kuosmanen and Post [16], who relax price certainty but maintain price exogeneity. Kuosmanen and Post obtain a similar reversal of the

involve a profit that falls short of $\frac{14}{3}$ at all possible prices. Clearly, this area is much smaller than the inefficient area in Figure 1. The fact that there are large white areas that are dominated in Pareto-Koopmans sense by firms 1 and 4 might at first seem counterintuitive. However, it reflects the fact that profits may actually decrease when netput amounts increase under endogenous prices. Recall that the traditional profit efficiency criterion classifies as inefficient any netput vector in the interior of com(*S*). That result does no longer apply if prices are endogenous; e.g. firm 2 lies in the interior of com(*S*) but it can not be classified as inefficient. The fact is that convexity and monotonicity are no longer harmless regularity properties in case of endogenous prices, as we discuss in the Section 5.

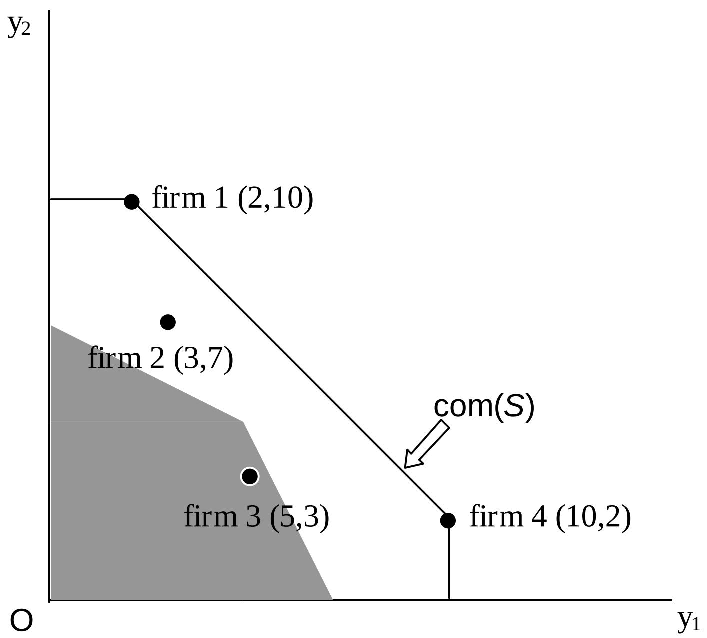

Fig. 2 Polyhedral reference set for generalized profit efficiency. Firm 3 is classified as inefficient; it achieves at all possible prices a profit that falls short of the minimum profit for firm 1 and firm 4.

'min' and 'max' operators. However, because exogenous prices are considered, uniform prices are used for the evaluated firm and the peers.

To illustrate the use of local price domain, we refine the example by dividing netput space $\mathbf{R}^2$ into two subsets $t_1 = \{(y_1, y_2) \in R^2 : y_1 \geq y_2\}$ and $t_2 = \{(y_1, y_2) \in R^2 : y_1 \leq y_2\}$ and set $A_1 = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$ and $b_1 = b_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$. The minimum profit level for firm 1 and firm 4 increases from $\frac{14}{3}$ to $\max_{y \in S}\left\{\min_{p \in \Pi(A,b)} \{py\}\right\} = \frac{18}{3}$. The gray area in Figure 3 represents all netput vectors that are classified as inefficient, i.e. that involve a profit that falls short of $\frac{18}{3}$ at all possible prices. Firm 3 remains profit inefficient because its maximum profit equals only $\frac{13}{3} < \frac{18}{3}$. In addition, firm 2 is also revealed as profit inefficient since the maximum profit amounts to $\frac{17}{3} < \frac{18}{3}$. This clearly illustrates how incorporating local price information can improve the power of our test.

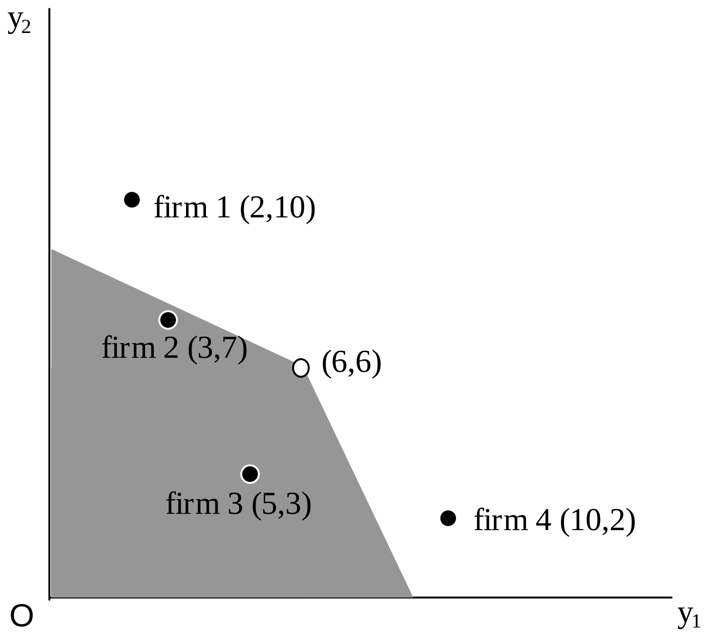

Fig. 3 Polyhedral reference set for generalized profit efficiency with the local price domains. Firm 2 is classified as inefficient; it achieves at all possible prices a profit that falls short of the minimum profit for the unobserved netput vector (6,6).

4. UNCERTAIN PRICES

As discussed in the Introduction, price endogeneity is often accompanied by price uncertainty. Kuosmanen and Post [16] develop a first-order stochastic dominance (FSD; Hadar and Russell [27]) test for the case of uncertain but exogenous prices. This section demonstrates that the test developed in Section 3 (which is developed to account for endogeneity rather than uncertainty) can also test efficiency in case of endogenous *and* uncertain prices.

To account for uncertainty, we further generalize our model of the firm; we summarize the beliefs of firm $j \in J$ about the prices by a subjective conditional distribution function $F(\cdot|y): \mathbf{R}_+^q \to [0,1]$, which assigns a probability to price vectors $p \in \mathbf{R}_+^q$ conditional upon the selected netput vector $y \in T$. The fact that we use a conditional distribution function to represent the price formation process allows for uncertain and endogenous prices. Obviously, price certainty (with inverse demand functions) forms a special case. We further assume that firm preferences can be represented in expected utility form, with a Von Neumann-Morgenstern utility function $U(\cdot): \mathbf{R}^1 \to \mathbf{R}^1$ that is monotone increasing in expected profit. This approach follows McCall [28] and Sandmo [29], among others; see Applebaum and Ullah [30] for further references. Still, the expected utility framework is used mainly for analytical convenience; the below FSD condition also applies for a whole range of non-expected utility theories of choice behavior under uncertainty (see e.g. Starmer [31]).

Under the above assumptions, a firm $j \in J$ is *profit efficient* if and only if

$$\pi^U(y_j, F, U, T) \equiv \max_{y \in T}\left\{ \int_{\rho \in \mathbf{R}_+^q} U(\rho y)\partial F(\rho|y) - \int_{\rho \in \mathbf{R}_+^q} U(\rho y_j)\partial F(\rho|y_j) \right\} = 0 . \qquad (7)$$

This test requires detailed information on *T*, *F* and *U*. In practice, such information typically is not available (and if it is available, then computational burden can exclude the application of this test). As discussed above, inner technology sets and outer price sets can reduce the requirement for production and price information (and reduce computational burden). In addition, we build on FSD to reduce the information requirement for the firm objective U (and to obtain a test that is computationally tractable). That criterion is very general; it is consistent with *all* monotone increasing U in the expected utility framework. We first define the conditional profit distribution function of firm $j \in J$ as

$$\Xi(\pi|y) \equiv \int_{\{p \in \Omega(y) | yp \leq \pi\}} \partial F(p|y) \quad \forall y \in \mathbf{R}^q . \qquad (8)$$

Formally, a distribution function $\Xi_1 : \mathbf{R}^1 \to [0,1]$ stochastically dominates another distribution function $\Xi_2 : \mathbf{R}^1 \to [0,1]$ by first-order, if and only if the former involves a probability of exceeding any value for z greater than or equal to that of the latter, i.e.

$$\Xi_2(\pi|\cdot) \succ \Xi_1(\pi|\cdot) \;\; \forall \pi \in \mathbf{R}^1, \qquad (9)$$

where $'\succ'$ denotes inequality $'\geq'$ with strict inequality $'>'$ holding for at least some value for π in the domain $\mathbf{R}^1$.

We are now equipped to generalize Theorem 2:

THEOREM 3 *If* $\Theta(S) \subseteq T$ *and* $\left\{\rho \in \mathbf{R}_{+}^{q} \middle| \delta F\left(\rho|y\right) > 0\right\} \subseteq \Omega(y) \;\; \forall y \in T$, *then* $\xi(y_j, \Omega, \Theta(S)) \leq 0$ *is a necessary FSD condition for* $\pi^{U}(y_j, F, U, T) = 0$.

PROOF If $\Theta(S) \subseteq T$ and $\left\{\rho \in \mathbf{R}_{+}^{q} \middle| \delta F\left(\rho|y\right) > 0\right\} \subseteq \Omega(y) \;\; \forall y \in T$, then $\xi(y_j, \Pi(A), \Theta(S)) > 0 \Leftrightarrow \exists y \in \Theta(S) : \min_{\rho \in \Pi(A)} \{\rho y\} > \max_{\rho \in \Pi(A)} \{\rho y_j\}$ implies $\exists y \in T : \min_{\rho \in P} \{\rho y\} > \max_{\rho \in P} \{\rho y_j\}$. This directly implies $\exists y \in T : \Xi\left(\pi|y_j\right) \succ \Xi\left(\pi|y\right) \;\; \forall \pi \in \mathbf{R}^{1}$, i.e. $\Xi\left(\pi|y\right)$ first-order stochastically dominates $\Xi\left(\pi|y_j\right)$ for some $y \in T$, for any specification of Ξ. Equivalently, $\pi^{U}(y_j, F, U, T) > 0$ for all F and monotone increasing U.■

5. ELICITING PRODUCTION AND PRICE INFORMATION

The power of the presented tests (i.e. the probability of Type II errors where the optimization hypothesis is erroneously maintained) generally depends on the tightness of the inner technology bound and the (local) outer price bound(s). This section discusses some alternative ways for eliciting and representing technology and price information, so as to improve the power of the tests.

PRODUCTION INFORMATION

As discussed in Section 2, the observed netput vectors are frequently complemented with assumptions about the structure of the production set like monotonicity, convexity and various types of returns to scale assumptions. These properties could be backed up e.g. by engineering knowledge or empirical evidence from the industry under evaluation. In addition, one can expand the sample *S* by introducing non-existing 'virtual firms' constructed e.g. by consulting experts of the field in the spirit of Thanassoulis and Allen [32], or by including observations from previous or subsequent time periods (after correcting for potential technological chance). Furthermore, knowledge of the sampling distribution can correct for small sample bias and can construct confidence intervals. Such knowledge can be obtained from the asymptotic sampling distribution (see e.g. Gijbels *et al.* [33]), or alternatively by means of bootstrapping (see e.g. Simar and Wilson [34]).

It is worth to emphasize at this point that the role of monotonicity and convexity assumptions under endogenous prices is very different from that in the traditional profit efficiency analysis. As discussed in Section 2, monotonicity and convexity assumptions are harmless when analyzing profit efficiency in the traditional sense, as in that case profit is linear and increasing in netputs. Many authors use this instrumental argument to motivate the use of $\mathrm{com}(S)$ (e.g. Varian [4] and Banker and Maindiratta [21]), rather than engineering knowledge or empirical evidence. Generally, monotonicity and convexity are harmlessly imposed on the constraint set if the profit function is a monotonically increasing and quasi-convex function of the netputs. For some settings it may be reasonable to assume monotonicity (e.g. when prices are 'well-behaved'; see Majumdar [35]) or quasiconvexity, even if quantities affect prices. However, there is no a priori reason for imposing these assumptions. For example, the $\min_{\rho \in \Pi(y)} \{\rho y\}$ component of our measure $\xi(y_j, \Pi, \Theta(S))$ generally is not increasing or quasiconvex, and therefore $\xi(y_j, \Pi, \mathrm{com}(\Theta(S)))$ generally exceeds $\xi(y_j, \Pi, \Theta(S))$.

Of course, monotonicity and convexity can be useful assumptions for increasing the power of the tests, provided there is a sound (theoretical, empirical or practical) reason for assuming the true production set would satisfy this property. For example, Figure 4 illustrates the effect of replacing S with $\mathrm{com}(S)$ in our example. The maximum profit increases from $\max_{y \in S} \left\{ \min_{\rho \in \Pi(A,b)} \{\rho y\} \right\} = \frac{14}{3}$ to $\max_{y \in \mathrm{com}(S)} \left\{ \min_{\rho \in \Pi(A,b)} \{\rho y\} \right\} = \frac{18}{3}$, achieved for the unobserved netput vector (6,6), constructed as an equally weighed average of firm 1 and firm 4. The shaded area represents the reference set. Interestingly, this area is exactly identical to the poyhedral reference set obtained by using local price domains in Figure 3. Again, firm 3 remains efficient, while firm 2 is classified as inefficient.

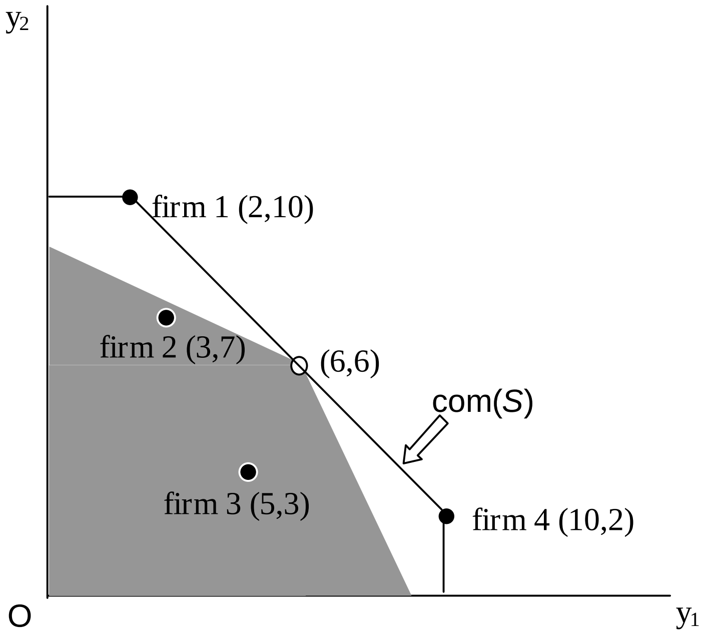

Fig. 4 Polyhedral reference set for generalized profit efficiency with the convex monotone hull as an empirical production set. Firm 2 is classified as inefficient; it achieves at all possible prices a profit that falls short of the minimum profit for the unobserved netput vector (6,6).

However, there is no a priori reason why technologies should be monotone or convex. For example, Farrell [36] stresses indivisible netputs and economies of scale and specialization as possible sources of non-convexities. In addition, Färe and Grosskopf [37] stress congestion of netputs as possible violation of monotonicity. McFadden ([38], pp. 9) explicitly states that the appeal of monotonicity and convexity assumptions in microeconomic production theory “lies in their analytical convenience rather than in their economic realism.”

We conclude that convexity and monotonicity assumptions introduce the risk of specification error in our generalized framework. For this reason, we propose to use empirical production sets that do not impose convexity or monotonicity when these assumptions can not be convincingly verified. One such production set is the observed set *S*. Conveniently, the measure $\xi(y_j, \Pi(A), S)$ can be computed by solving *n*+1 linear programming problems for each firm to be evaluated. Specifically, we can compute the measure from the maximum of the solutions to the *n* linear programming problems $\min_{\rho\in\Pi(y_i)}\{\rho y_i\}$ $\forall i \in J$ and the solution to the linear programming problem $\max_{\rho\in\Pi(y_j)}\{\rho y_j\}$. This provides an additional, practical argument for using *S* as an empirical production set.

PRICE INFORMATION

Information for the construction of (generic or local) price domains may come from various sources. In many cases, economic theory can guide in the specification of price restrictions. For example, Kuosmanen and Post [39] used the theoretical insight that the cost of equity capital exceeds that of debt capital (because equity involves more risk for the capital suppliers than debt does) for measuring the economic efficiency of European banks. Similarly, one could use the stylized fact that the wage rate for white-collar workers is higher than that for blue-collar workers, even though the wage difference would be dependent on the supply or demand forces. Further, one can exploit empirical knowledge of the industry under evaluation. For example, the application in Section 6 restricts the electricity prices using the observed trading range on the Amsterdam Power Exchange (APX). In addition, one can use information concerning written contracts, professional agreements or government rules that regulate quantity and price reactions; see e.g. our discussion of the 'protocol' of the Dutch electricity sector in Section 6. Finally, price information can be elicited by interacting with firm managers or industry experts e.g. using interactive tools from the Multi-Criteria Decision-Making literature (see e.g. Halme *et al.* [40]; compare also with Thanassoulis and Allen [32], who propose similar techniques for exploiting local production information).

A convenient simplification occurs if we partition the netput space into the n+2 exclusive and exhaustive subsets $t_j = y_j \quad \forall j \in J$, $t_{n+1} = \{y \in \Theta(S) : y \notin S\}$, $t_{n+2} = \{y \in \mathbf{R}^q : y \notin \Theta(S)\}$. We can then specify $\Pi(A_j, b_j)$ for each observed $j \in J$ and leave $\Pi(A_{n+1}, b_{n+1})$ and $\Pi(A_{n+2}, b_{n+2})$ void (or, if reasonable, set $\Pi(A_{n+1}, b_{n+1}) = \bigcup_{j \in J} \Pi(A_j, b_j)$). Constructing cells (centered at the observations) that include all netput vectors for which the price bounds are assumed to hold could refine this approach. This approach is especially attractive if information is available on the prices associated with the actual netput choices, and if there is a possibility to interact with the firm managers on their perception of the price formation process and the implication of changes to the actual netput choices.

To illustrate this latter approach, we recapture the earlier example. We divide netput space $\mathbf{R}^2$ into five subsets $t_j = y_j$ ($j = 1,2,3,4$; subscripts refer to the respective firms) and $t_5 = \{y \in \mathbf{R}^2 : y \notin S\}$, and suppose $A_1 = \begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$, $A_3 = A_4 = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$ and $b_1 = b_2 = b_3 = b_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$, while the price domain is left unspecified for the other possible netput allocations (i.e. A_5, b_5 void). The minimum profit level for firm 1 and firm 4 remains $\frac{14}{3}$. Further, firm 3 remains profit inefficient. However, also firm 2 will now be revealed as profit inefficient since the maximum profit amounts to

$\frac{13}{3}$ only. This clearly illustrates how incorporating local price information can improve the power of the proposed profit efficiency test.

6. EMPIRICAL APPLICATION

Non-parametric efficiency analysis has seen numerous applications for regulating the utilities in various countries. For example, the regulator of the Dutch electricity sector recently applied the approach for a system of price cap regulation. For the purpose of illustration, we performed an application of our approach to data for the Dutch electricity sector in 2000.

We start with a brief description of the Dutch electricity sector, and its different components: production, transmission and distribution. Electricity *production* in the Netherlands has traditionally been dominated by four major power producing companies: EPON, UNA, EPZ, and EZH account for approximately 60 billion KWh, which is roughly 60 percent of the total power output (roughly 100 billion KWh). The remaining 40 percent are accounted for by co-generated power produced by large industrial users. Domestic production is supplemented by imports from Belgium, France, Germany and Norway. Imports currently account for roughly 10 percent of the total electricity supply. Electricity *transmission* is split between the national transmission operator (TenneT) for the national 220/380 kV network high-voltage network and regional operators for transmission up to 150 kV. Finally, 18 regional electricity *distribution* companies distribute electricity to roughly 7 million electricity consumers. These distribution companies are vertically integrated with the regional network operators.

In this study, we analyze the regional distribution and network companies (henceforth DNCs). For the sake of simplicity, we use a simplified representation of the production technology that involves a single output, the number of customers (Cust), and two inputs, (1) operating expenses (Opex; materials, services, wages and other costs) in thousands of Euro and (2) the electricity purchases in thousands of megawatt hours (MWh).[7] Table 1 lists the full data set.

Table 1

DNC	**Cust**	**Opex** (x 1000 Euro)	**MWh** (x 1000)
1	45388	7801	466
2	1073193	191537	11261
3	86325	8567	862
4	494391	103000	4608
5	2656133	515455	8065
6	193880	48400	7059
7	43714	11750	828
8	98418	11305	862
9	11560	2642	70
10	49743	6250	433
11	30012	2831	272
12	111189	16206	1034
13	37942	5262	291
14	19496	2900	206
15	938369	192388	13409
16	888336	199183	10256
17	408635	113520	9051
18	45117	7167	335

We stress that this application is for illustrative purposes only. A more realistic representation of the production technology would account for differences across DNCs in e.g. quality of service, customer mix, geography, and network architecture. In addition, a sound study would improve the power of the tests by using additional data e.g. from time series or from international comparisons. For these reasons, this application can not assess whether or not the Dutch DNCs are truly efficient, nor measure the true degree of efficiency.

The electricity markets in the European Union are currently in a process of transformation to a liberalized market for production and distribution. In 2000, the Dutch electricity market had made some important steps towards liberalization. Electricity production was fully liberalized (and the foreign companies PreussenElektra, Electrabel and Reliant Energy acquired EZH,

[7] For a full description of the variables and the DNCs in our model, we refer to the homepage of the regulatory office for the Dutch electricity sector (http://www.dte.nl).

EPON, and UNA respectively). However, the market for consumer distribution still involved regional monopolies regulated by a system of price caps. Since DNCs have a legal obligation to serve all customers in their service area, and in addition price caps regulate prices, it makes sense to assume that output quantities and prices were fixed. By contrast, input prices were not fixed. In fact, the local market conditions suggest that prices were endogenous and uncertain in a non-trivial way. The capacity to import electricity from neighboring countries was limited, and the Dutch electricity market was a domestic market. In addition, the domestic market was highly concentrated, as there were only four producers, and the 18 distributors were effectively integrated into a small number of larger entities (and 3 large companies remain: Essent, NUON, and ENECO). Finally, a long-run agreement between producers and distributors (the so-called 'protocol') reduced the liquidity on the Amsterdam Power Exchange (APX). Under these circumstances, it seems reasonable to assume that individual firms could affect prices in a non-trivial way. This assumption is supported by the ENECO case that made headline news during the winter of 1999-2000. ENECO planned to import a substantial amount of electricity from abroad, and hence they limited the amount contracted with the domestic producers. However, they had overestimated the import capacity allocated to them by TenneT, and they still had to rely on the domestic market. Since the 'protocol' included a penalty price of 600 Euro/MWh (i.e. 125% of 'normal' prices), prices on the APX exploded. The affair blew over when the minister of Economic Affairs intervened and the 'protocol' was renegotiated. However, the case clearly demonstrates that input prices were not exogenous and certain.

For these reasons, it makes sense to develop a model where the DNCs face exogenous and certain output quantities and prices, but endogenous and uncertain electricity prices. Therefore, we tested, for each DNC, whether the data set contains another DNC that served more customers than the evaluated DNC, and that had costs (Opex plus energy costs) that exceed at all possible prices the cost of the evaluated DNC. Based on the 'protocol' agreements and the historical APX prices, it seems reasonable to assume that the average price paid by DNCs ranged between 20 and 50 Euro/MWh.[8] Therefore, we solved for each DNC the following problem:

$$(1) \qquad \min_{\substack{i\in\{1,\cdots,18\}: \\ \mathrm{Cust}_i \geq \mathrm{Cust}_j}} \left\{ \max_{\rho\in[20,50]} \left\{ \mathrm{Opex}_i + \rho \mathrm{MWh}_i \right\} \right\} - \min_{\rho\in[20,50]} \left(\mathrm{Opex}_j + \rho \mathrm{MWh}_j \right).$$

All DNCs passed the test, except for DNC 7. DNC 18 served more customers than DNC 7 (see Table 1). In addition, at the maximum price of 50 Euro/MWh, DNC 18 involves Opex and energy cost of approximately 23 million Euro. By contrast, at the minimum price of 20 Euro/MWh, DNC 7 involves Opex and energy cost of approximately 28 million Euro. Hence, DNC 7 necessarily achieves a lower profit than DNC 18, and failed to optimize production, even if we account for the possibility of endogenous and uncertain prices.

Of course, our approach usually involves less discriminating power than the traditional approach (especially if price and technology information is limited, as in this study). To illustrate this point, assume for simplicity that the standard model would hold and that DNCs minimize costs at given price of say 35 Euro/MWh. In this case, 7 out of 18 DNCs are classified as inefficient (see table 2). This may be the result of the power of the traditional test. However, it could also reflect that the traditional test does not account for market power or uncertain, e.g. evaluating cost efficiency at a single electricity price (*in casu*: 35 Euro/MWh) does not capture the complete price formation process faced by the firms. In any event, this application demonstrates that ignoring price endogeneity and uncertain may have a dramatic influence on the results.

[8] Historical data on APX indexes is available at http://www.apx.nl.

Table 2

DNC	**Generalized cost efficiency** 1=efficient, 0=inefficient	**Traditional cost efficiency** 1=efficient, 0=inefficient
1	1	0
2	1	1
3	1	1
4	1	1
5	1	1
6	1	1
7	0	0
8	1	1
9	1	1
10	1	1
11	1	1
12	1	1
13	1	1
14	1	0
15	1	0
16	1	0
17	1	0
18	1	0

In this application we have used a generic price domain. As discussed in Section 3 the discriminating power of the efficiency tests can be increased by using local price domains. As this application merely serves illustrative purposes, we will not explore that possibility in full in this section. However, we point out that the Dutch ' protocol' includes penalty rates for consuming more electricity than initially contracted. Hence, one could refine our analysis e.g. by using a low fixed price up to the contracted amount and a high variable price for excess demand.

7. DISCUSSION

We have derived a test for optimizing behavior in case of endogenous prices, witch holds under both price certainty and price uncertainty. The test is fully consistent with the original non-parametric orientation, because it does not require detailed assumptions about the market structure. In addition, the test is compatible with the existing tools for representing technology and price information, and preserves the tractable mathematical programming structure of the original methodology.

By building on a minimal set of maintained assumptions, we minimize Type I errors, i.e. the probability that the optimization hypothesis is wrongly rejected. In this respect, violations of our test provide strong evidence against optimizing behavior. However, there still are alternative interpretations for such violations:

1. Our maintained assumptions may be wrong. We have focused on a minimal set of maintained assumptions, and (when compared to the conventional approach) we excluded imperfect competition as a possible source of error. Nonetheless, some of our assumptions may still lead to erroneous rejections of the profit maximization hypothesis. We see at least the following sources of Type I errors:

    A. Our tests are motivated by the fact that it is difficult in many research environments to accurately approximate the production technology and the prices (or indirect demand functions). Still, we adhered to the standard assumption that the observed production vectors are feasible (i.e. $S \subseteq T$). Unfortunately, the observations sometimes are contaminated by errors-in-variables, e.g. because different firms use different valuation and depreciation schemes (or simply because of typing errors). We could therefore combine our generalized framework with the established tools to account for statistical significance (Varian [41]; and Matzkin [42]) and economic significance (Varian [43]) of violations.

    B. We may doubt whether firms are really interested in maximizing profits when price endogeneity prevails. While in a neoclassical framework maximum profit is usually accepted without question as the right objective for a firm, matters become more complicated under imperfect competition, as already pointed out by Marshall ([44], p. 402). The main argument is that firm owners are not interested in monetary profit as such but rather in its purchasing power, and that owner preferences as consumer may interfere with owner preferences as producer. Dierker and Grodal [45] recently provided an exhaustive discussion of this issue. Nevertheless, profit maximization is generally maintained as a behavioral assumption when modeling firm behavior under endogenous prices (see e.g. Hart, [13]). Also for this reason, it is interesting to expose the profit maximization hypothesis under price endogeneity to empirical tests.

C. We relaxed the assumptions of exogenous and certain prices. However, we adhered to a deterministic technology. That is, we have assumed throughout that the output amounts produced and the input quantities consumed by firms were perfectly certain. For some industries (e.g. agriculture), this is not a very realistic assumption. This calls for developing testing tools that take such quantity uncertainty explicitly into account. Such tools could for example be constructed along similar lines as those followed in this paper. For example, the argument would be straightforwardly reversed if prices were the perfectly controllable decision variables and netput quantities the uncertain random variables.

2. On the other hand, when we could reasonably conjecture that our maintained assumptions (accurate measurement, profit maximization and exogenous prices) hold, remaining violations can be interpreted as truly profit inefficient behavior (compare with Banker and Maindiratta, [21]). Such firm-level inefficiency could for example be rationalized by relating it to agency problems within the firm. That is, the firm may not act in accordance with profit maximization because the firm managers, who pursue different goals, are not fully controlled by the firm owners. In effect, negative test results do not immediately indicate "irrational" behavior, as the firm managers may act rational. Rather, they suggest inconsistency of firm behavior with the owner preferences. In this respect, it seems worthwhile to explore whether and to what extent the testing tools could be employed as monitoring instruments by the firm owners (compare with Bogetoft [46], [47]).

In the introduction we noted that easy extensions of the proposed tools can recover production and price information. Similarly, the non-parametric tools can be used for testing and recovering information about the endogenous price system (e.g. about the inverse demand functions; compare with Varian [4], Section 10). As we have indicated, additional price and production information entails more powerful efficiency tests and so stronger results of the efficiency analysis.

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