

# StoNED

## Stochastic Nonparametric Envelopment of Data Cross-sectional Frontier Estimation Subject to Shape Constraints

**StoNED model** combines the nonparametric DEA frontier with the stochastic noise term of SFA, melding the virtues of both approaches into a unified framework of productive efficiency analysis. Both DEA and SFA are constrained special cases of StoNED, as evident from the table below

**Stage 2:** Given the NLS residuals  $\varepsilon$ , estimate  $\sigma_v$  and  $\sigma_u$  by

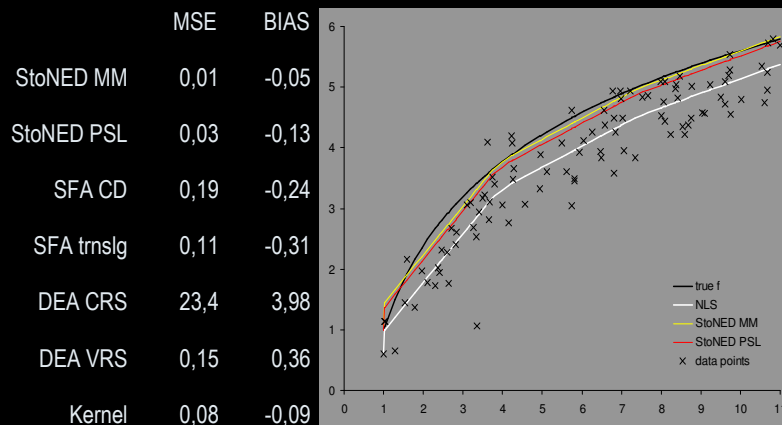
- method of moments (MM)
- maximum pseudolikelihood (PSL)

Compute the conditional expected values of inefficiency:  $E(u_i|\varepsilon_i)$  and  $E(E(u_i)|\varepsilon)$ .

	parametric (Xb)	nonparametric (f)
deterministic	COLS $y = Xb - u$ $u \geq 0$	DEA $y = f(X) - u$ $u \geq 0$ $f$ monotone, concave
stochastic	SFA $y = Xb - u + v$ $u \sim N(0, \sigma_u), v \sim N(0, \sigma_v)$	StoNED $y = f(X) - u + v$ $u \sim N(0, \sigma_u), v \sim N(0, \sigma_v)$ $f$ monotone, concave

Legend:  $y$  = output vector,  $X$  = input matrix,  $b$  = parameter vector,  $f$  = production function,  $v$  = noise terms,  $u$  = inefficiency terms [note:  $Eff(x_i, y_i) = y_i / (y_i + u_i)$ ].

**Example** with simulated data of 100 observations from production function  $y = \ln(x) + 1$



**Estimation** of StoNED model involves 2 stages:

**Stage 1:** Nonparametric least squares (NLS) subject to monotonicity and concavity constraints

$$\min_{\varepsilon, \alpha, \beta} \sum_{i=1}^n \varepsilon_i^2$$

$$y_i = \alpha_i + \beta'_i x_i + \varepsilon_i$$

$$\alpha_i + \beta'_i x_i \leq \alpha_h + \beta'_h x_i \quad \forall h, i = 1, \dots, n$$

$$\beta_i \geq 0 \quad \forall i = 1, \dots, n$$

### Additional features and extensions

- returns to scale (VRS, CRS, NIRS, NDRS)
- multiplicative errors [ $y = f(X) / (1 - u + v)$ ]
- cost functions, distance functions, etc.
- statistical inference by bootstrapping
- panel data models (fixed and random effects)

### Further information and working papers:

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