Cost efficiency analysis of electricity distribution networks: Application of the StoNED method in the Finnish regulatory model

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Abstract
Electricity distribution network is a prime example of a natural local monopoly. In many countries, electricity distribution firms are regulated by the government. In Finland, the regulator estimates the efficient cost frontier using the data envelopment analysis (DEA) and stochastic frontier analysis (SFA) methods. This paper reports the main results of the research project commissioned by the Finnish regulator for further development of the efficiency estimation in their regulatory model. The key objectives of the project were to integrate a stochastic SFA-style noise term to the nonparametric, axiomatic DEA-style cost frontier, and take into account the heterogeneity of firms and their operating environments. To estimate the resulting stochastic semi-parametric cost frontier model, a new method called stochastic nonparametric envelopment of data (StoNED) is proposed. Based on the insights obtained in the empirical analysis using real data of the regulated networks, replacing the currently used DEA and SFA methods by the StoNED method is recommended.

Key words:
Energy markets; heterogeneity; nonparametric production analysis; productive efficiency
1. Introduction

Distribution of electricity to end users forms the final stage of the supply network of electricity. The distribution networks usually take power from high voltage transmission lines and distribute it to electricity consuming firms and households. Since construction of competing networks in the same area is usually prohibitively expensive, the distribution networks enjoy a natural monopoly within their established distribution area. In many countries, the national or local governments have established regulatory agencies to monitor the electricity distribution networks and counter the abuse of local monopoly power. Thus, the immediate challenge faced by the regulators is to determine an acceptable price for electricity distribution in a sector characterized a heterogeneous group of firms operating in heterogeneous environments. In the long run, a related challenge is to provide incentives for improving productivity and adopting the best technologies and practices.

The regulation of electricity distribution networks typically involves both static and dynamic cost efficiency analysis of the distribution firms. This is one of the most significant application areas of the productive efficiency analysis in terms of the policy relevance and economic implications. The frontier estimation techniques such as data envelopment analysis (DEA; Farrell 1975; Charnes et al., 1978) and stochastic frontier analysis (SFA; Aigner et al., 1977; Meeusen and vanden Broeck, 1977) are widely employed by regulatory agencies around the world. Some of the recent published applications include Pahwa et al. (2003) [USA], Jamasb and Pollit (2003) [Europe], Edvardsen and Forsund (2003) [Nordic countries], Estache et al. (2004) [South America], Filippini et al. (2004) [Slovenia], Thakur et al. (2005) [India], von Hirschhausen et al. (2006) [Germany], Cullmann and von Hirschhausen (2008) [Poland], and Kopsakangas-Savolainen and Svento (2008) [Finland].

Nordic countries have a particularly strong tradition in the applications of the frontier estimation techniques to the regulation of electricity distribution (Hjalmarsson and Veiderpass, 1992; Forsund and Kittelsen, 1998; Agrell et al., 2005). In Finland, the Energy Market Authority (Energiamarkkinanvirasto, henceforth referred to as EMV) has applied the DEA method since 1998. The landmark study by Korhonen, Syrjänen and Tötterström (2000) [a shorter version published in English in Korhonen and Syrjänen, 2003], defined the input and output variables and the main axioms of the DEA-based cost frontier
that are still applied by EMV in its current regulatory model. Another significant development in the evolution of the Finnish model was the study by Syrjänen, Bogetoft and Agrell (2006), which recommended the adoption of SFA as a parallel frontier estimation technique in addition to DEA. In years 2008-2011 the regulatory model of EMV sets the efficiency improvement targets based on the arithmetic average of the firm-specific DEA and SFA efficiency estimates. The purpose of using the average efficiency estimate is to reduce the statistical uncertainty related to both DEA and SFA methods and their underlying assumptions.

As a logical next step in the evolution of the Finnish regulatory model, EMV commissioned a research project to investigate how the essential characteristics of the DEA and SFA methods could be integrated in the Finnish regulatory model by employing a unified stochastic frontier method called StoNED (stochastic nonparametric envelopment of data), developed in Kuosmanen (2006, 2008) and Kuosmanen and Kortelainen (2007). The StoNED method combines the non-parametric, piece-wise linear DEA-style frontier with the stochastic SFA-style treatment of inefficiency and noise. The assumptions of the StoNED method are milder than those required by DEA or SFA: both DEA and SFA can be obtained as constrained special cases of the more general StoNED-model (Kuosmanen and Johnson, 2010). This makes the StoNED method more robust to the statistical uncertainty concerning the exact functional form of the cost frontier and the impacts of stochastic noise than the classic DEA and SFA methods.

This paper reports the main results of the project (the final report by Kuosmanen et al. 2010 contains additional material but it is only available in Finnish). We believe the findings of this study are very interesting both for the research community in the field of productive efficiency analysis and for the regulatory agencies in other countries. This study demonstrates the practical benefits of the StoNED method over the classic DEA and SFA approaches in a real world regulatory application. While this study focuses on the electricity distribution networks, the novel analytical techniques employed in this study are broadly applicable to the regulation of natural monopolies and networks not only in energy sector but in other industries as well.

The rest of the paper is organized as follows. Section 2 introduces the cost frontier model and its maintained assumptions. Section 3 presents the StoNED method for estimating the functions and parameters of interest. Section 4 briefly describes the data. In Section 5 we present the main empirical
results based on our preferred model specification. In Section 6 we report some specification tests and sensitivity analyses based on alternative model specifications and assumptions. Section 7 presents the concluding remarks and the policy recommendations. Additional 3-D graphical illustrations of the StoNED-frontier are provided in the Appendix.

2. Cost frontier model

The cost frontier model employed by EMV uses the total cost as the single aggregate input factor. The total cost of firm \( i \) is denoted by \( x_i \), and it includes the total capital expenditure, controllable operational expenditure, and the cost of interruptions. The three output variables \( (y) \) specified in the model are: \( y_1 \) is the weighted amount of energy transmitted through the network (GWh of 0.4 kV equivalents), \( y_2 \) is the total length of the network (km), and \( y_3 \) is the total number of customers connected to the network. In output \( y_1 \), the transmission of electricity at different voltage levels is weighted according to the average cost of transmission such that the high-voltage transmission gets a lower weight than the low-voltage transmission. The specification of the input and output variables is based on EMV’s current regulatory model and it is commonly used in the literature (see e.g. Korhonen and Syrjanen, 2003; Thakur et al., 2007).

In addition to the outputs \( y \), the proportion of underground cables in the total length of the network is taken into account as a contextual variable \( z \). The rationale of using an additional \( z \) variable that is not an input or output as such is to better control for the heterogeneity of the firms and their operating environments. In Finland, underground cables are widely used in the cities and suburban areas, but the overhead power-lines remain a more economical technology in the rural and sparsely populated areas. By using the proportion of underground cables as a contextual variable we take into account the higher cost of underground cables, but this is not the only purpose of this variable. More importantly, we are also trying to capture a range of other effects of urban versus rural operating environment on firm performance. For example, the average wage rate is generally higher in large cities and towns than in rural areas where the employment opportunities are scarce and thus the opportunity
cost of labor is lower. Thus, the contextual variable \( z \) is likely capture the statistical effect of higher wage rate on total cost, among many other sources of heterogeneity.

The cost frontier model considered in this study can be analytically presented as

\[
x_i = C(y_i) \cdot \exp(\delta z_i + u_i + v_i),
\]

where \( C \) is the frontier cost function, \( \delta \) is a parameter that characterizes the effect of underground cables \( z \) on firm’s total cost, \( u_i \) is a random variable that represents cost inefficiency of firm \( i \), and \( v_i \) is a stochastic noise term that captures the effects of measurement errors, omitted variables and other random disturbances to the otherwise stable cost equation.

We do not assume any specific functional form for the cost function \( C \). Similar to the classic DEA (Charnes et al., 1978), we impose the following axioms:

1) \( C \) is monotonic increasing in all outputs \( y \) (the sub-gradients satisfy \( \nabla C(y) \succeq 0 \ \forall y \)).

2) \( C \) is globally convex (\( C(\lambda y_1 + (1-\lambda)y_2) \geq C(\lambda y_1) + C((1-\lambda)y_2) \ \forall \lambda > 0; y_1, y_2 \in \mathbb{R}^3 \)).

3) \( C \) exhibits constant returns to scale (CRS) (\( C(\lambda y) = \lambda C(y) \ \forall \lambda > 0 \)).

The random variables \( u \) and \( v \) are assumed to independently distributed of each other, and of outputs \( y \) and contextual variable \( z \). Following the classic SFA studies (Aigner et al. 1977), the noise term \( v \) is assumed to be normally distributed with a zero mean a finite variance \( \sigma^2_v > 0 \) and the inefficiency term \( u \) follows the half-normal distribution with a finite variance \( \sigma^2_u > 0 \). The expected value of inefficiency is denoted by \( E(u_i) = \mu \), and it is known to be directly proportional to the parameter \( \sigma_u \):

\[
\mu = \sigma_u \sqrt{2/\pi} \quad (Aigner et al. 1977).
\]

This model is more general in its assumptions than the classic DEA and SFA models currently employed by EMV. It is easy to verify that the original DEA specification by Charnes et al. (1978) is obtained as the restricted special case of the above model if we set \( \sigma^2_v = 0 \) (no noise) and \( \delta = 0 \) (exclude the contextual variable). Similarly, the linear SFA model proposed by Syrjänen et al. (2006) is obtained by imposing the additional parametric assumption that cost frontier \( C \) is a linear function.
Thus, both DEA and SFA specifications are restricted special cases of the more general model assumed in this paper.

EMV currently sets the efficiency improvement targets based on the arithmetic average of the DEA and SFA efficiency estimates. The main purpose of this procedure is to reduce the statistical uncertainty related to both DEA and SFA estimators and their underlying assumptions. However, the statistical properties of the two estimators are only known under the very restrictive conditions where the assumptions of both DEA and SFA hold simultaneously (i.e., the cost function is linear and there is no noise). Under these assumptions the SFA estimator is unbiased, consistent, and asymptotically efficient, whereas the DEA estimator is consistent but biased. In the ideal case, the SFA estimator is more efficient than the arithmetic average of DEA and SFA. Unfortunately, neither DEA nor SFA are very robust to the violation of their basic assumption. There is no evidence that averaging the DEA and SFA estimators improve the precision of the estimator if the assumptions of either one of the methods - or both - fail to hold. A method that builds upon a more general set of assumptions is the logical way forward.

3. StoNED method

In this section we present the StoNED method (stochastic nonparametric envelopment of data) for estimating the general cost frontier model introduced in the previous section. In contrast to the classic DEA and SFA approaches, the StoNED method does not require any additional assumptions or parameter restrictions. The StoNED method has two steps:

1) estimation of the expected total costs and the parameter $\delta$ by nonparametric least squares
2) estimation of the expected inefficiency, variance parameters, and firm-specific inefficiencies

First, without loss of generality, we can introduce a composite error term $\varepsilon_i = u_i + v_i$, and linearize the cost frontier model by taking natural logarithms of both sides of the equation to obtain

$$\ln x_i = \ln C(y_i) + \delta z_i + \varepsilon_i.$$
This gives a semiparametric, partial linear model to be estimated. Convex nonparametric least squares (CNLS: Kuosmanen, 2008; Johnson and Kuosmanen, 2009) regression provides consistent estimator for the expected value of the total cost $x_i$ and the parameter $\delta$. The CNLS estimator is obtained as the optimal solution to the following least squares problem, which can be solved by convex programming algorithms and solvers

$$
\min_{\gamma, \beta, \delta, \sigma, \epsilon} \sum_{i=1}^{n} \epsilon_i^2 \\
\text{s.t.} \\
\ln x_i = \ln \gamma_i + \delta z_i + \epsilon_i \quad \forall i \\
\gamma_i = \beta_i y_i \quad \forall i \\
\gamma_i \geq \beta_i y_i \quad \forall h, i \\
\beta_i \geq 0 \quad \forall i
$$

Coefficients $\beta_i$ can be interpreted as marginal costs of outputs or as the coefficients of the tangent hyperplanes to the piece-wise linear cost frontier. These coefficients are analogous to the multiplier weights in DEA. In contrast to the linear regression model, the coefficients $\beta_i$ are specific to each firm. This is an important feature for modeling heterogeneity of the distribution networks: urban distribution networks with a large number of customers are generally assigned a higher marginal cost for output $y_3$ than the rural networks, for which the length of the network (output $y_2$) is a more important cost driver.

Parameter $\gamma_i$ is the CNLS estimator of the expected total cost of producing $y_i$, that is, $E(x_i) = C(y_i) + \mu$. To estimate the cost frontier, we need to estimate $\mu$ that remains unknown after the first step. To this end, we can utilize the distribution of the CNLS residuals $\hat{\epsilon}_i$, obtained in the optimal solution to the CNLS problem (see Kuosmanen and Kortelainen, 2007, for details). Under the maintained assumptions of half-normal inefficiency and normal noise, the second and third central moments of the composite error distribution are given by

$$
M_2 = \left[ \frac{\pi - 2}{\pi} \right] \sigma_2^2 + \sigma_\epsilon^2, \\
M_3 = \left[ \frac{2}{\pi} \right] \left[ \frac{4}{\pi} - 1 \right] \sigma_\epsilon^3.
$$

These central moments can be estimated by using the CNLS residuals:
\[ M_2 = \frac{1}{n} \sum_{i=1}^{n} (\hat{e}_i - \overline{e})^2 / n, \quad M_3 = \frac{1}{n} \sum_{i=1}^{n} (\hat{e}_i - \overline{e})^3 / n. \]

Note that the third moment \( M_3 \) (which measures the skewness of the distribution) only depends on the standard deviation parameter \( \sigma_u \) of the inefficiency distribution. Thus, given the estimated \( \hat{M}_3 \) (which should be positive in the case of a cost frontier), we can estimate \( \sigma_u \) parameter by

\[ \hat{\sigma}_u = \sqrt{\frac{M_3}{2 \left( \frac{2}{\sqrt{\pi}} - \frac{4}{\pi} - 1 \right)}}. \]

Subsequently, the standard deviation of the error term \( \sigma_v \) is estimated using

\[ \hat{\sigma}_v = \sqrt{\hat{M}_2 - \frac{\pi - 2}{\pi} \hat{\sigma}_u^2}. \]

These MM estimators are unbiased and consistent (Aigner et al., 1977; Greene, 2008).

The cost function \( C \) is estimated by

\[ \hat{C}_{\text{StoNED}}(y) = \gamma_1 \cdot \exp\left(-\hat{\sigma}_u \sqrt{2/\pi}\right). \]

Since both \( \gamma_1 \) and \( \hat{\sigma}_u \) are statistically consistent, the estimator \( \hat{C}_{\text{StoNED}}(y) \) is also consistent. In practice, the StoNED cost frontier is obtained by shifting the CNLS estimate of the average-practice production function upwards by the expected value of the inefficiency term, analogous to the MOLS approach (e.g., Greene, 2008).

Firm-specific inefficiency components \( u_i \) must be inferred indirectly in the cross-sectional setting. Jondrow et al. (1982) have shown that the conditional distribution of inefficiency \( u_i \) given \( e_i \) is a zero-truncated normal distribution with mean \( \mu_u = -e_i \sigma_u^2 / (\sigma_u^2 + \sigma_v^2) \) and variance \( \sigma_u^2 = \sigma_u^2 \sigma_v^2 / (\sigma_u^2 + \sigma_v^2) \). As a point estimator of \( u_i \), one can use the conditional mean

\[ E(u_i \mid e_i) = \mu_u + \sigma_u \left[ \frac{\phi(-\mu_u / \sigma_u)}{1 - \Phi(-\mu_u / \sigma_u)} \right], \]

where \( \phi \) is the standard normal density function, and \( \Phi \) is the standard normal cumulative distribution function (see Kuosmanen and Kortelainen, 2007, for details). The conditional expected
value is an unbiased but inconsistent estimator of \( u_i \); irrespective of the sample size \( n \), we have only a single observation of the firm \( i \) (see, e.g., Greene, 2008, Section 2.8.2, for further discussion). The Jondrow et al. (1982) estimates \( \hat{u}_i \) can be converted to cost efficiency measures \( (CE) \) expressed in the percentage scale by using

\[
CE = 100\% \times \exp(\hat{u}_i).
\]

The range of the cost efficiency scores \( CE \) is \([0\%, 100\%]\), where \( CE = 100\% \) corresponds to cost efficient activity level.

Finally, Johnson and Kuosmanen (2009) show that the CNLS estimator of the parameter \( \delta \) is unbiased, consistent, asymptotically normal and efficient, and converges at the standard parametric rate of order \( n^{-1/2} \). Thus, the essential statistical properties of the parametric estimators carry over to the parametric part of the semiparametric estimator, even though the cost frontier \( C \) is estimated in a fully nonparametric fashion. This enables us to apply conventional methods of inferences for testing the statistical significance of the contextual variable \( z \).

### 4. Data

The total cost \( x \), the three input variables \( y \), and the contextual variable \( z \) were introduced in Section 2. As the regulatory model is based on four-year periods, EMV applies the average total cost and the average output levels in the previous four-year period as the input-output variables of the cross-sectional cost frontier model. In the similar fashion, the empirical data used in this study consists of four-year averages over the period 2005-2008. Prior to averaging, all cost variables have been deflated to the price level of 2005 using the price index of the construction sector published by the Statistics Finland.

Table 1 reports descriptive statistics of the observed \( x \), \( y \) and \( z \) variables. The total cost of the sector was approximately 750 Million € per year during the study period (at the prices of year 2005). The average total cost was approximately 8.5 Million €, but the maximum was as high as 117.5 Million €. The total cost of the two largest distributors (Fortum Sähkönsiirto Oy and Vattenfall Verkko Oy)
forms approximately 30% of the total cost of the sector, while a large majority of the companies are small local operators. This shows as relatively large skewness in all variables reported in Table 1.

Table 1: descriptive statistics of the input, output, and contextual variables (four-year averages of period 2005-2008).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Kurtosis</th>
<th>Skewness</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = Total cost (1,000 €)</td>
<td>8,418.91</td>
<td>18,047.78</td>
<td>25.04</td>
<td>4.75</td>
<td>267.81</td>
<td>117,554.10</td>
</tr>
<tr>
<td>y₁ = Energy transmission (GWh)</td>
<td>480.39</td>
<td>971.51</td>
<td>22.41</td>
<td>4.41</td>
<td>14.81</td>
<td>6,599.71</td>
</tr>
<tr>
<td>y₂ = Length of network (km)</td>
<td>4,135.27</td>
<td>10,223.27</td>
<td>26.88</td>
<td>4.99</td>
<td>50.80</td>
<td>67,611.05</td>
</tr>
<tr>
<td>y₃ = No. customers</td>
<td>35,448.68</td>
<td>71,870.65</td>
<td>17.08</td>
<td>3.98</td>
<td>24.25</td>
<td>420,473.00</td>
</tr>
<tr>
<td>z = Proportion of underground cables in the total network length</td>
<td>0.33</td>
<td>0.26</td>
<td>-0.52</td>
<td>0.73</td>
<td>0.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The amount of energy transmission $y₁$ measures the direct (variable) output of the distribution activity. In contrast, outputs $y₂$ and $y₃$ represent the potential output or the capacity. It is worth to note that distribution of electricity to recreational homes forms a significant proportion of activity for the companies located in rural areas. Recreational homes consume relatively small amount of electricity, especially in winter months when recreational homes are not used. However, the network companies have a legal obligation to connect all customers within their designated area, and the firms have to maintain the power lines in operation even if they are only used seasonally, which can be relatively costly. To take this into account, outputs $y₂$ and $y₃$ capture the fixed cost of maintaining a sufficient capacity to serve their designated network area irrespective of the actual consumption of power.

Several earlier studies have used the peak load (e.g., the maximum transmission of energy within a period of one hour) or the load factor (i.e., the ratio of peak load to average load) as output variables that measure the network capacity (e.g., Kopsakangas-Savolainen and Svento, 2008). However, in the present data set the peak load correlates almost perfectly with the output variable $y₁$: the correlation coefficient of the peak load and the average load is $r = 0.996$. Therefore, introducing the peak load as an output variable would cause serious multicollinearity problems for the estimation. Further, introducing additional output variables affects the precision of the semi- and non-parametric estimators due to the curse of dimensionality. The present set of output variables effectively captures
the costs associated with the peak load through the high statistical correlation with the output variable \( y_1 \) even though the peak load is not explicitly included in the model.

To some extent, the three output variables can take into account heterogeneity of firms and their operating environments. In general, networks located in rural areas have a long network \( (y_2) \) relative to the number of customers \( (y_3) \), whereas urban networks have a relatively short network. Thus, the ratio \( y_3 / y_2 \) captures reasonably well the heterogeneity of urban versus rural networks. Note that in the StoNED method the marginal costs of outputs can differ across firms, so the performance of rural networks can be best rationalized by assigning a relatively large marginal cost for the network length and a small marginal cost for the number of customers. In contrast, urban networks are evaluated in the most favorable light by assigning large marginal cost on the number of customers and a small marginal cost on the network length. The logic of assigning the marginal costs follows directly analogous to the shadow pricing approach of the classic DEA.

While the output variables \( y_2 \) and \( y_3 \) can draw a distinction between urban versus rural networks, the networks located in large cities have very similar output structure as those located in suburbs or small towns. To better capture the differences in the output structures and operating environments of the urban versus suburban networks, we have introduced the proportion of underground cables as a \( z \) variable that represents the net impact of practices and operational conditions on the total cost. In this study the proportion of underground cables is calculated based on the total network length. Alternative \( z \) variables as well as alternative definitions of the proportion of underground cables (e.g., the proportion of underground cables in the medium-voltage network) were considered. The model specification reported in this paper was chosen based on the empirical fit of the model \( (R^2) \), taken into account the economic and statistical significance of the \( z \) variables considered. Note that some earlier studies have similarly used the proportion of underground cables as an explanatory variable in the second-stage regression where DEA efficiency scores are regressed on \( z \) variables. In contrast to those studies, the coefficient of the \( z \) variable is here jointly estimated together with the nonparametric cost frontier. This integrated estimation procedure allows us to avoid the
omitted variable bias of the efficiency estimation (if z variable is ignored in the first stage efficiency analysis), and the standard techniques of statistical inferences are applicable (see Johnson and Kuosmanen, 2009, for details).

5. Results
The estimated StoNED-frontier is graphically illustrated in Figure 1. The three-dimensional diagram displays the output set at the fixed cost level of one Million €. The three axes of the diagram represent the three output variables: energy transmission, length of network, and number of customers. The piece-wise linear boundary presented in Figure 1 describes the output combinations that can be produced with the total expenditure of one Million €. Of course, output combinations below the efficient frontier are also feasible according to the free disposability assumption. The origin of the output set lies below the frontier: Figure 1 displays the estimated output frontier from outside.

To gain intuition, we have indicated the approximate locations of rural, suburban, urban, and industrial networks in Figure 1. Rural networks have relatively large length of the network, but low energy transmission and small number of customers. These networks are located in the bottom right corner of Figure 1. When we move towards suburban and urban networks, the number of customers and energy transmission increase while the length of network decreases. Networks operating in large cities are found in the top left corner of Figure 1. The data set also includes a small number of industrial networks that supply energy to a small number of large manufacturing plants. These industrial networks have a relatively large amount of energy transmission, whereas the length of network and the number of customers can be very small. The industrial networks are found on the top of the diagram. In some industrial towns the heavy industry can consume a large proportion of energy distributed through the network. The output profiles of the networks located in industrial towns are somewhere between the large cities (left corner) and the industrial networks (top corner) in the figure.
Some hyperplane segments of the output frontier are clearly visible in Figure 1, but a large number of segments are visually indistinguishable in the diagram. The total number of hyperplane segments is 77. As we move from the rural networks via suburban and urban networks to the industrial networks, we can observe a boomerang shaped edge of the frontier that represents the efficient subset of the frontier. This boomerang shaped subset consists of several dozens of small hyperplane segments that have very similar slopes. Interestingly, 57% of distribution networks operate in this narrow, boomerang shaped part of the frontier. This narrow efficient subset of the frontier is where the marginal costs of the outputs are determined.

There are 21 weakly efficient hyperplane segments where the marginal cost of at least one output is equal to zero. 43% of firms operate in these weakly efficient segments. Almost all of them are
located in the large weakly efficient triangle on the right side of the diagram where the marginal cost of
the number of customers is equal to zero. These firms are typically small networks operating in the
rural areas, with a relatively small number of customers. Only one network operates in the smaller
weakly efficient subset where the marginal cost of the network length is zero (on the left side of the
diagram). None of the distribution firms operate in the dark colored vertical part of the frontier where
the marginal cost of electricity transmission is equal to zero.

As the StoNED method is based on least squares regression, the empirical fit of the model
can be measured by using the conventional coefficient of determination (R^2). As the model has been
specified in the log-linear form, the coefficient of determination measures the proportion of the
variance of natural logarithm of total costs (ln x) explained by the model. The statistic is defined as

\[ R^2 = 1 - \frac{\text{Est Var}(\hat{e})}{\text{Est Var}(\ln x)} \]

where

- \( \text{Est Var}(\hat{e}) \) is the sample variance of the CNLS residuals,
- \( \text{Est Var}(\ln x) \) is the sample variance of the total cost x.

The coefficient of determination obtained by the previous formula is 0.9864. This means that
the StoNED cost frontier explains over 98 percent of the observed variation in the natural logarithm of
total cost across the networks. The R^2 statistic includes the effects of three outputs \( y_1, y_2, y_3 \), the
contextual variable \( z \), and the expected value of the inefficiency term \( u \) The 1.36% of variation that
remains unexplained by the model includes the deviations of the inefficiency term \( u \) from its mean, and
the noise term \( v \). By construction, the StoNED frontier maximizes the empirical fit under the
postulated axioms: no other cost function \( C \) that satisfies free disposability, convexity and constant
returns to scale can achieve a higher R^2 statistic.

As the cost function is assumed to exhibit constant returns to scale, the shape of the output
sets remains the same at all non-negative cost levels. That is, the diagram presented in Figure 1 applies
to different cost levels if we simply rescale the axes. For example, at the total cost of 10 Mill. €, the numbers on the three axes should be multiplied by factor 10, but otherwise the same diagram applies.

Figure 1 reveals the complexity of the estimation problem. If we estimate the cost function by the linear SFA model (currently applied by EMV), the results will be driven by the large weakly efficient triangle on the right of the diagram. However, the relevant marginal costs of outputs are determined on the boomerang shaped efficient set of Figure 1. Clearly, the cost structure of rural networks is very different from those of the urban or industrial networks. The linear SFA model applies the same marginal costs to all distribution networks, whereas the StoNED model takes into account the heterogeneity of the firms and their cost structures into account by allowing the marginal costs differ across firms depending on their output structure. To assess all networks in the most favorable light, the StoNED method assigns urban networks a relatively large marginal cost to the number of customers, whereas the rural networks are assigned a relatively large marginal cost to the length of network.

Table 2 reports some descriptive statistics of the distribution of the estimated marginal costs. These marginal costs are obtained by multiplying the shadow prices $\beta$ obtained as the optimal solution to the CNLS problem by the expected value of inefficiency $\mu$. Although the shadow prices $\beta$ are firm-specific, in practice, the shadow prices tend to be heavily clustered. For 33 networks the shadow prices are close to the median values reported in Table 2. On the other hand, it is worth to note that the shadow prices are not necessarily unique (analogous to DEA). From Figure 1 it is evident that for the firms located in the vertices or edges of the piecewise linear frontier it is impossible to determine any unique tangent hyperplane. There are as many as 39 such firms in our sample. For these firms we have estimated the marginal costs as the arithmetic average of all adjacent efficient and weakly efficient hyperplane segments.

It is interesting to compare the results of Table 2 with the marginal cost estimates obtained by linear regression. The OLS estimates of the marginal costs are 0.28 c/GWh for energy transmission, 895 €/km for the length of network, and 95 €/user for the number of customers. These estimates are of similar magnitude as the firm-specific StoNED estimates reported in Table 2. However, according to
the StoNED estimates, the marginal costs of energy transmission and the network length appears to be somewhat higher for most firms. On the other hand, the marginal cost of the customers is lower even for the networks located in large cities.

| Table 2: Descriptive statistics of the distribution of the estimated firm-specific marginal costs of outputs |
|----------------------------------|-----------------|-----------------|-----------------|
|                                | Energy transmission (c/kWh) | Length of network (€/km) | Number of customers (€/customer) |
| Average                        | 0.477            | 930.09          | 12.94           |
| St. dev.                       | 0.122            | 172.09          | 18.33           |
| Median                         | 0.5686           | 985.44          | 0.00            |
| Mode                           | 0.6072           | 912.12          | 0.00            |
| Min                             | 0.0518           | 0.00            | 0.00            |
| Max                             | 0.6126           | 1045.25         | 76.36           |

The proposed model specification includes the proportion of underground cables in the total length of network as a z variable that represents the operating environment of the network. The z variable determines the level of the cost frontier, but does not influence its shape. In the present model specification, it is possible to interpret the z variable as a factor that explains the observed efficiency differences across firms, or alternatively, as a variable that represents the heterogeneity of the firms and their operating environments. Both interpretations are equally plausible, and we do not find any good reason to prefer one over another. We are mainly interested in isolating the effect of the z variable from the unexplained differences in the total costs of the firms, so that the efficiency improvement targets imposed on the firms take the operating environment better into account.

We estimate the effect of the z variable by using the one-stage semi-parametric method of Johnson and Kuosmanen (2009). One of the advantages of this approach is that the conventional methods of statistical inferences apply to the coefficient δ of the z variable. Table 3 reports the parameter estimate of the coefficient δ and the related regression statistics. We find that the proportion of underground cables has a highly significant positive effect on the total cost (p-value 0.0000). Noting that \( \exp(0.36) = 1.43 \), we find that the total cost of a network using 100% of underground cables has on
average 43% higher total cost than an identical network using overhead cables. The z variable explains approximately 30% of the differences in the logarithm of cost efficiency \([\ln \text{TOTEX}_i - \ln C(y)_i]\) across networks (the partial \(R^2\) statistic reported on the bottom row of Table 3). It is worth emphasizing that the z variable does not only capture the direct cost of using underground cables, but also other factors correlated with the z variable. Since underground cables are mainly used in urban and suburban areas, the statistical correlation between the total cost and the z variable will also capture such effects as the higher wage rate in the cities, which has nothing to do with the physical construction or maintenance of the distribution network. We interpret the z variable as a proxy for the urban operating environment. It is difficult to find exact measures of the operating environment, but we find that a large proportion of otherwise unexplained cost differences across firms can be explained by the differences in the utilization of underground cables.

Table 3: Parameter estimates of the z variable (proportion of underground cables in the total length of network).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta) coefficient</td>
<td>0.3600</td>
</tr>
<tr>
<td>standard error</td>
<td>0.0581</td>
</tr>
<tr>
<td>t-statistic</td>
<td>6.1942</td>
</tr>
<tr>
<td>p-value</td>
<td>0.0000</td>
</tr>
<tr>
<td>95% lower limit</td>
<td>0.2443</td>
</tr>
<tr>
<td>95% upper limit</td>
<td>0.4752</td>
</tr>
<tr>
<td>partial (R^2)</td>
<td>0.3060</td>
</tr>
</tbody>
</table>

Cost efficiency was estimated using the method of moments estimator applied to the CNLS residuals as described in Section 3. The estimated variance parameters of the inefficiency and noise terms and the expected values of inefficiency are reported in Table 4. The expected value of cost efficiency is 89%. This expected value applies to all distribution networks. Firm-specific efficiency estimates can be obtained by applying the conditional expected valued derived by Jondrow et al. (1982). The arithmetic average of the firm-specific conditional efficiency estimates is 92%, somewhat higher than the unconditional expected value.
Table 4: Parameter estimates related to the inefficiency and the noise terms and the expected value of inefficiency

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$ (variance of the composite error term)</td>
<td>0.03239</td>
</tr>
<tr>
<td>$\sigma_c^2$ (variance of the inefficiency term)</td>
<td>0.02064</td>
</tr>
<tr>
<td>$\sigma_v^2$ (variance of the noise term)</td>
<td>0.01175</td>
</tr>
<tr>
<td>$\mu$ (expected value of the inefficiency term)</td>
<td>0.11464</td>
</tr>
<tr>
<td>Expected value of cost efficiency</td>
<td>89%</td>
</tr>
</tbody>
</table>

Firm-specific efficiency estimates are based on confidential data, and hence cannot be reported in this study. To illustrate the distribution of firm-specific estimates, we have plotted the cumulative frequency distribution of the cost efficiency estimates in Figure 2. The plotted curve indicates the proportion of distribution networks (on the vertical axis) that achieves at least the cost efficiency level indicated on the horizontal axis (or smaller). For example, 30% of networks operate with cost efficiency of 90% or less. In other words, 70% of the firms achieve cost efficiency of 90% or higher. More than 40% of firms operate with cost efficiency of 95% or higher. According to our estimates, a large majority of the distribution network operate with high degree of cost efficiency. Unfortunately there are some networks that operate with 80% cost efficiency or lower. In order to assign more stringent efficiency improvement targets to the least efficient companies, it is crucially important to be able to isolate inefficiency from the heterogeneity represented by the $z$ variable and from stochastic noise.
In Table 5 the distribution networks have been classified to 10 groups based on the estimated marginal costs of outputs. The groups have been sorted in descending order according to the marginal cost of the energy transmission (the marginal costs presented in the table are the group averages). Firms in the first groups are typically rural networks with a relatively small number of users relative to the network length. Urban networks are classified in groups 7 and 10 (group 10 includes urban firms, with a very heterogeneous set of marginal costs; the average marginal costs of this group are left unreported). The rightmost column reports the average cost efficiency of the group. Differences in the average efficiency across groups are remarkably small. This suggests that the estimated StoNED frontier captures the heterogeneous cost structures reasonably well, and provides a fair and equitable benchmark to rural, suburban as well as urban networks.

The marginal costs reported in Table 5 present the performance of each firm in the most favorable light. Suppose the regulator allows the firms to freely choose in which group they want to be classified. If the firms are rational and maximize their efficiency score, all firms would choose to be
classified in the same group as the StoNED method has indicated. In other words, no firm has an incentive to deviate from the group it has been assigned in Table 5.

Table 5: Classification of networks to 10 groups according to the estimated marginal costs.

<table>
<thead>
<tr>
<th>Group</th>
<th>No. of firms</th>
<th>Energy transm. (c/kWh)</th>
<th>Length of network (€/km)</th>
<th>No. of users (€/user)</th>
<th>Average efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>0.6043</td>
<td>876.74</td>
<td>0.87</td>
<td>92%</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>0.5597</td>
<td>904.94</td>
<td>1.23</td>
<td>92%</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.4434</td>
<td>908.77</td>
<td>22.25</td>
<td>94%</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.4566</td>
<td>1038.81</td>
<td>1.86</td>
<td>93%</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.4200</td>
<td>970.69</td>
<td>21.00</td>
<td>92%</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0.3662</td>
<td>964.71</td>
<td>27.86</td>
<td>95%</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>0.2929</td>
<td>232.21</td>
<td>60.11</td>
<td>92%</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>0.3493</td>
<td>930.93</td>
<td>33.43</td>
<td>91%</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>0.3324</td>
<td>983.05</td>
<td>29.61</td>
<td>90%</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.4773</td>
<td>930.09</td>
<td>12.94</td>
<td>92%</td>
</tr>
</tbody>
</table>

Although the StoNED method is computationally more demanding than the conventional DEA and SFA methods, once the frontier has been estimated, it can be presented in a relatively simple piece-wise linear form. As a part of the project, we developed a simple Excel application that can be used for computing the StoNED efficiency scores and cost targets for any arbitrary values of outputs y and the contextual variable z. This application enables both the regulator and the regulated firms make own calculations and analyses within the framework of the model specification provided in Section 2 (data y, z can be changed, but changes to the model specification are not possible in the Excel application). The Excel application is applicable for most modeling purposes of the regulator and the network firms. For example, it is possible to evaluate the impacts of increasing demand for electricity in the future on the level of acceptable total cost and efficiency. The regulator can also use the application for computing the acceptable total cost of each network in the future to anticipate the impacts of the regulatory model and to implement the model. An English version of the Excel application is available free of charge at the StoNED website: http://www.nomepre.net/stoned/.
6. Specification tests and sensitivity analysis

6.1 Returns to scale

The StoNED method enables one to test or postulate various specifications regarding the returns to scale (RTS), similar to DEA. The recommended model specification described in Section 2 is based on the assumption of constant returns to scale (CRS) that does not allow any premium or advantage for firms operating at small or large scale. In this sense, the CRS specification treats all firms equally, using the most productive scale size as a common benchmark to firms of all sizes.

The regulatory model currently used in Finland is based on the assumption of non-decreasing returns to scale (NDRS). This specification allows small firms to appeal to a disadvantage due to the small scale, which is compensated in the model by allowing the small firms include a cost premium. However, large firms are not allowed to have a premium for the diseconomies of scale. The assumption of NDRS can be justified if the cost function exhibits economies of scale and if the firms cannot influence the scale. In practice, however, a number of mergers and divisions to smaller companies have occurred, so the scale of operation is not exogenously given to the firms. If small companies are allowed a scale premium, it might give the wrong incentives for the firms to avoid mergers that would improve overall cost efficiency, or even encourage firms to split into smaller and more cost inefficient companies in order to take advantage of the small-scale premium. From the perspective of regulation, the specification of the returns to scale should take into account two questions: 1) Does the technology exhibit economies or diseconomies of scale? 2) Can the firms influence the scale of their operations by mergers or divisions?

To address the first question, we estimated the cost frontier by the StoNED method under CRS, NDRS, and variable returns to scale (VRS) specifications. By comparing the distributions of the regression residuals under the three alternative RTS specifications, we can assess if the returns to scale specification influences the efficiency estimates. Further, we can test if the impact is statistically significant.
Table 6 summarizes some of the results obtained under three alternative RTS specifications. The impact on empirical fit (coefficient of determination $R^2$) is very small. The coefficient of underground cables is slightly higher in the NDRS and VRS specifications. Table 6 also reports the correlation coefficients of the CNLS residuals across the three alternative RTS specifications. We find that the residuals of the VRS and NDRS specifications are almost perfectly correlated. The correlation between CRS and VRS residuals is high, almost 0.9. This suggests that the RTS specification has relatively minor influence on the shape of the cost frontier or the efficiency estimates.

<table>
<thead>
<tr>
<th></th>
<th>CRS</th>
<th>NDRS</th>
<th>VRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.9864</td>
<td>0.9894</td>
<td>0.9895</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.3600</td>
<td>0.4162</td>
<td>0.4178</td>
</tr>
<tr>
<td>Correlation of residuals</td>
<td>1</td>
<td>0.8994</td>
<td>0.8958</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.9967</td>
</tr>
</tbody>
</table>

Significance of the RTS specification can be statistically tested by applying the nonparametric Kolmogorov-Smirnov (KS) test (Banker, 1993; Banker and Natarajan, 2004). The KS-test does not require any particular distributional assumptions, which is a desirable property because the CNLS residuals include a normally distributed noise term $v$, an asymmetric inefficiency term $u$, and possible errors of specification and sampling error. The null hypothesis of the KS test is that the cost frontier exhibits CRS. The alternative hypothesis can be VRS or NDRS. If the alternative RTS specification yields significantly different results, the null hypothesis is rejected. The test statistic of the KS test is 0.0787, irrespective of whether we test CRS against VRS or NDRS specification. This value of the test statistic is very small and not significant at the conventional significance levels. The $p$-value of the KS test is as high as 0.937. Thus, we cannot reject the null hypothesis of CRS in the KS-test.

We could similarly test if there are significant differences in the frequency distributions of the efficiency estimates under different RTS specifications. Unfortunately, the CNLS residuals have a negatively skewed distribution in the VRS and NDRS specifications, which violates the basic
assumption of the cost frontier model where the asymmetry of the inefficiency term implies the residuals of a cost frontier should be positively skewed. The wrong skewness occurs frequently in the estimation of stochastic frontier models, irrespective of whether one uses conventional SFA or the proposed StoNED method, or whether one uses a maximum likelihood or least squares estimator. According to Greene (2008), the wrong skewness can be seen as a built-in diagnostic test: wrong skewness may be a sign of model misspecification. In the present setting, imposing VRS or NDRS specification if the underlying technology exhibits CRS could explain the wrong skewness. On the other hand, Monte Carlo simulations suggest that the wrong skewness occurs with a relatively high frequency also in correctly specified models. In any case, it is impossible to estimate the cost frontier or the efficiency estimates under the VRS and NDRS specifications, so we resort to the CRS specification.

As noted before, the StoNED cost frontier under CRS specification explains more than 98% of the variation in the logarithm of total cost across the firms. Relaxing the CRS assumption improves the empirical fit only about 0.3 percentage points. Applying the conventional F-test, it is clear that such a marginal increase in the empirical fit is statistically insignificant even if it cost only one additional degree of freedom (in fact, the VRS specification involves 89 additional parameters). Regarding the economic significance, the estimated cost due to the diseconomies of scale amounts to 7.7 Million € in the NDRS specification. For comparison, the total cost of all firms was 747 Million, so the economic significance of the RTS specification is only approximately 1 percent of the total cost.

Finally, we can assess whether firms operating with different scale have some advantage or disadvantage by regressing the inefficiency estimates $u$ obtained in the CRS specification on the three outputs $y_1$, $y_2$, and $y_3$. The regression results are reported in Table 7 below. We find the coefficients of all three outputs are close to zero and statistically insignificant. The model as a whole is not jointly significant according to the F-test.

In conclusion, we cannot reject the CRS hypothesis in the statistical KS-test. Further, the size of the firm does not explain the efficiency differences across firms. Therefore, we conclude that the
CRS specification provides a fair and equitable benchmark to all firms, and does not systematically favor either small or large distribution networks.

Table 7: Regression of StoNED inefficiency estimates $u$ (under CRS) on outputs $y_1, y_2, y_3$.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t test</th>
<th>p-value</th>
<th>95% lower bound</th>
<th>95% upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.089248</td>
<td>0.009075</td>
<td>9.833940</td>
<td>0.000000</td>
<td>0.071203</td>
</tr>
<tr>
<td>$y_1$ (energy)</td>
<td>-0.000018</td>
<td>0.000056</td>
<td>-0.327681</td>
<td>0.743959</td>
<td>-0.000129</td>
</tr>
<tr>
<td>$y_2$ (length)</td>
<td>0.000000</td>
<td>0.000002</td>
<td>0.290812</td>
<td>0.771904</td>
<td>-0.000003</td>
</tr>
<tr>
<td>$y_3$ (no. users)</td>
<td>-0.000000</td>
<td>0.000001</td>
<td>-0.009826</td>
<td>0.992183</td>
<td>-0.000001</td>
</tr>
</tbody>
</table>

$R^2$ | 0.034052 |
F-test stat | 0.998812 | 0.397518 |

6.2 Heteroskedasticity

Following the classic SFA studies (e.g., Aigner et al., 1977), we have assumed that the inefficiency and noise terms are homoskedastic, and hence have the same finite variance irrespective of the scale size. The scale of operation is one of the most common sources of heteroskedasticity: larger firms have typically larger variations in inefficiency and noise than small firms. Note that the multiplicative, log-linear specification of the cost frontier takes this effectively into account: even though the random variables $u$ and $v$ are assumed to have constant variances, the variance of total cost $x$ increases as outputs $y$ increase. Multiplicative specification of the disturbance term is commonly used technique to alleviate heteroskedasticity due to the scale of activity.

Heteroskedasticity can cause various problems that are well known in econometrics. The ordinary least squares estimator is unbiased and consistent despite heteroskedasticity, but it becomes potentially inefficient. In addition, heteroskedasticity affects the standard errors, which should be taken into account when drawing statistical inferences. Estimation of the inefficiency term and its expected value can be sensitive to heteroskedasticity. Therefore, it is advisable to test if the CNLS residuals show any signs of heteroskedasticity.

The standard White (1980) test of heteroskedasticity can be applied. This test is particularly suited for the StoNED method because it is based on least squares residuals and does not assume any
particular model or specification of heteroskedasticity. The null hypothesis of the White test is homoskedasticity. The test is based on an auxiliary regression where we regress the squared residuals on outputs $y$, the squared outputs, and their cross-products. If the auxiliary regression can explain the squared residuals, the null hypothesis is rejected. The White test statistic is defined as $nR^2$. Under the null hypothesis, this test statistic has the $\chi^2$ distribution with the degrees of freedom equal to the number of slope coefficients in the auxiliary regression model (here 9).

The results of the auxiliary regression are reported in Table 8. The empirical fit of the model is poor, and the null hypothesis of homoskedasticity cannot be rejected. The p-value of the White test is 0.1908. All estimated coefficients of the auxiliary regression are very close to zero, and none of the coefficients is statistically significant. Based on these results, we conclude that the multiplicative specification of the inefficiency and noise terms seems an effective method to capture the scale effect on the variance of total cost. There are no signs of heteroskedasticity in the CNLS residuals.

Table 8: White test of heteroskedasticity: auxiliary regression of squared CNLS residuals

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard error</th>
<th>t test</th>
<th>p-value</th>
<th>95% lower limit</th>
<th>95% up. limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.02330994</td>
<td>0.00365391</td>
<td>6.37945896</td>
<td>0.00000001</td>
<td>0.01603703</td>
</tr>
<tr>
<td>$y_1$</td>
<td>0.00002317</td>
<td>0.00003720</td>
<td>0.62280247</td>
<td>0.53520808</td>
<td>-0.00005088</td>
</tr>
<tr>
<td>$y_2$</td>
<td>-0.00000186</td>
<td>0.00000217</td>
<td>-0.85516659</td>
<td>0.39504505</td>
<td>-0.00000618</td>
</tr>
<tr>
<td>$y_3$</td>
<td>-0.00000036</td>
<td>0.00000055</td>
<td>-0.65661224</td>
<td>0.51333846</td>
<td>-0.00000145</td>
</tr>
<tr>
<td>$y_1^2$</td>
<td>0.00000013</td>
<td>0.00000008</td>
<td>1.55179174</td>
<td>0.12470920</td>
<td>-0.00000004</td>
</tr>
<tr>
<td>$y_2^2$</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>0.73985628</td>
<td>0.46158022</td>
<td>-0.00000000</td>
</tr>
<tr>
<td>$y_3^2$</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>1.36147090</td>
<td>0.17723537</td>
<td>-0.00000000</td>
</tr>
<tr>
<td>$y_1 \times y_1$</td>
<td>-0.00000000</td>
<td>0.00000000</td>
<td>-1.23166070</td>
<td>0.22172957</td>
<td>-0.00000001</td>
</tr>
<tr>
<td>$y_2 \times y_3$</td>
<td>0.00000000</td>
<td>0.00000000</td>
<td>0.82568616</td>
<td>0.41146869</td>
<td>-0.00000000</td>
</tr>
<tr>
<td>$y_1 \times y_3$</td>
<td>-0.00000000</td>
<td>0.00000000</td>
<td>-1.41976830</td>
<td>0.15960933</td>
<td>-0.00000001</td>
</tr>
</tbody>
</table>

$R^2 = 0.13901004$

F-test = 1.41720497

White test stat ($nR^2$) = 12.3718936

6.3 Industrial networks

In Finland, all distribution networks are subject to regulation. However, networks that significantly differ from their peers in terms of the scale size, production or cost structure, or technology utilized can be excluded from the efficiency analysis. Earlier studies conducted in Finland have excluded three
so-called industrial networks that mainly focus on distributing power to heavy manufacturing. In Figure 1 these three firms are located on the top of the frontier, with a relatively large amount of distributed energy, short network length, and small number of customers. In the deterministic DEA method such atypical observations with a special output profile can have a major influence on the shape of the cost frontier and the efficiency estimates.

To assess the sensitivity of the StoNED estimates on the impacts of three industrial networks, we have estimated the recommended model specification with and without the three industrial networks. For other firms, the correlation of the CNLS residuals between the two model specifications is very high (0.9933). Correlation of the inefficiency estimates $u$ is almost as high (0.9927). Average cost efficiency of other networks is 91.11% when the industrial networks are excluded from analysis. When the industrial networks are included, the average efficiency increases to 91.36%.

In conclusion, the StoNED cost frontier and the related efficiency estimates are not particularly sensitive to inclusion or exclusion of the three industrial networks, which have a rather atypical output structure. Interestingly, other networks are slightly better off if the industrial networks are included in the analysis than if they are excluded. By investigating the efficiency estimates, we find that one of the industrial networks is highly efficient, whereas the other two industrial networks are relatively inefficient. On average, cost efficiency of industrial networks slightly lower than that of the other distribution networks. This explains why excluding the industrial networks does not increase the estimated efficiency of other networks.

In contrast to DEA that estimates the frontier based on a relatively small number of extreme observations, the StoNED frontier utilizes information of all $n$ observations in the sample. This is why the impacts of leaving out a single observation, even a firm with an unusual output structure, are likely to be small. The relative robustness of the StoNED method to individual observations is an attractive property in the regulation of a sector where regulated firms merge or split.
7. Conclusions and policy recommendations

Based on the results of the project, the final report by Kuosmanen et al. (2010) recommends the Finnish regulator EMV to replace the currently used DEA and SFA method by the cost frontier estimated using the StoNED method. The key advantages of the StoNED method include:

1) The stochastic noise is modeled explicitly in a probabilistic manner.
2) Heterogeneity of the firms and their operating environments is taken into account.
3) Conventional statistical tests and confidence intervals can be applied.
4) The method is relatively user-friendly compared to other semi-parametric alternatives.

In the following we briefly elaborate the previous four points.

1) The StoNED method is based on regression analysis, specifically, convex nonparametric least squares. The stochastic noise is attributed to the regression residual similar to the conventional regression analysis. As Kuosmanen and Johnson (2010) have shown, the classic DEA estimator is obtained as a constrained special case of the convex nonparametric least squares regression. Therefore, the DEA and SFA methods currently used by EMV are both restricted special cases of the more general StoNED method.

2) Observed heterogeneity of firms and their operating environments can be taken into account in the StoNED model by using contextual variables. Johnson and Kuosmanen (2009) have shown that the estimated coefficients of contextual variables are statistically consistent, asymptotically efficient, asymptotically normally distributed, and converge at the same parametric rate as the conventional regression coefficients. Thus, the curse of dimensionality that causes problems for the inference in the second stage regression of conventional DEA efficiency estimates is not a problem in the StoNED method: precision of the estimator is not affected even if one uses several contextual variables to describe the operational conditions or practices of the evaluated firms.

3) The probabilistic nature of the StoNED method makes reliable statistical inferences relatively easy. For example, confidence intervals of the firm specific efficiency scores can be estimated analogous to the SFA method. Further, it is possible to test for the statistical significance of the
efficiency differences across firms or groups of firms (Kuosmanen and Fosgerau, 2009). Statistical significance of the contextual variables can be tested by the conventional t-test, without computationally demanding bootstrap simulations. Specification tests such as testing the returns to scale specification are also available (see Section 6.1). In regulatory applications, reliable statistical tests are critically important for justifying the model specification and efficiency estimates used in the real-world regulation.

4) StoNED method builds directly upon the basic assumptions and axioms of DEA and SFA, and it does not require any additional assumptions or tools. This makes the StoNED method relatively user-friendly compared to other semi- and non-parametric alternatives available in the literature. The StoNED method does not require computationally intensive bootstrap simulations or kernel estimation, and it is conceptually easy to master based on the conventional DEA and SFA. Solving the nonparametric least squares problem requires sufficient hardware resources and a proper solver for quadratic or nonlinear programming, but once the StoNED frontier has been estimated, computing cost targets or reference points on the frontier is easy. As a part of the project we developed a simple Excel-spreadsheet to enable EMV and the regulated power companies make their own efficiency analyses using the estimated StoNED frontier. One can insert existing or forecasted values of outputs and costs, and the Excel-spreadsheet will automatically compute the efficient cost level and the efficiency improvement target in percents and Euro based on the StoNED cost frontier and the inserted cost.

In conclusion, the method proves well suited the purposes of the cost efficiency estimation in the regulatory model that EMV applies in Finland. It provides several distinct advantages compared to the conventional DEA and SFA methods currently used. We believe the StoNED method could be an interesting alternative for the regulatory applications world-wide, not only in electricity distribution but in the regulation of local monopolies in general.
References


Appendix: Three-dimensional illustrations of the estimated StoNED frontier

Figure A.1: Output set of the estimated StoNED frontier at the total cost of 1 Milj. €; three dimensional illustrations from four different angles.